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THE MONIST

ON THE NATURE OF ACQUAINTANCE.

II.

NEUTRAL MONISM.

"NEUTRAL MONISM"—as opposed to idealistic monism and materialistic monism—is the theory that the things commonly regarded as mental and the things commonly regarded as physical do not differ in respect of any intrinsic property possessed by the one set and not by the other, but differ only in respect of arrangement and context. The theory may be illustrated by comparison with a postal directory, in which the same names come twice over, once in alphabetical and once in geographical order; we may compare the alphabetical order to the mental, and the geographical order to the physical. The affinities of a given thing are quite different in the two orders, and its causes and effects obey different laws. Two objects may be connected in the mental world by the association of ideas, and in the physical world by the law of gravitation. The whole context of an object is so different in the mental order from what it is in the physical order that the object itself is thought to be duplicated, and in the mental order it is called an "idea," namely the idea of the same object in the physical order. But this duplication is a mistake: "ideas" of chairs and tables are identical with chairs and tables, but are

considered in their mental context, not in the context of physics.

Just as every man in the directory has two kinds of neighbors, namely alphabetical neighbors and geographical neighbors, so every object will lie at the intersection of two causal series with different laws, namely the mental series and the physical series. "Thoughts" are not different in substance from "things"; the stream of my thoughts is a stream of things, namely of the things which I should commonly be said to be thinking *of*; what leads to its being called a stream of *thoughts* is merely that the laws of succession are different from the physical laws. In my mind, Cæsar may call up Charlemagne, whereas in the physical world the two were widely sundered. The whole duality of mind and matter, according to this theory, is a mistake; there is only one kind of *stuff* out of which the world is made, and this stuff is called mental in one arrangement, physical in the other.[1]

A few quotations may serve to make the position clearer. Mach says (*op. cit.*, p. 14):

"That traditional gulf between physical and psychological research, accordingly, exists only for the habitual stereotyped method of observation. A color is a physical object so long as we consider its dependence upon its luminous source, upon other colors, upon heat, upon space, and so forth. Regarding, however, its dependence upon the retina...., it becomes a psychological object, a sensation. Not the subject, but the direction of our investigation, is different in the two domains."

"The primary fact is not the *I*, the ego, but the ele-

[1] For statements of this theory, see William James, *Essays in Radical Empiricism*, Longmans, 1912, especially the first of these essays, "Does 'Consciousness' Exist?" See also Mach, *Analysis of the Sensations*, Chicago, 1897 (the original was published in 1886). Mach's theory seems to be substantially the same as James's; but so far as I know James does not refer to him on this subject, so that he must have reached his conclusions independently of Mach. The same theory is advocated in Perry's *Present Philosophical Tendencies* and in *The New Realism* (1912).

ments (sensations). The elements *constitute* the *I*. *I* have the sensation green, signifies that the element green occurs in a given complex of other elements (sensations, memories). When *I* cease to have the sensation green, when *I* die, then the elements no longer occur in their ordinary, familiar way of association. That is all. Only an ideal mental-economical unity, not a real unity, has ceased to exist.

"If a knowledge of the connection of the elements does not suffice us, and we ask, *Who* possesses this connection of sensations, *Who* experiences the sensations, then we have succumbed to the habit of subsuming every element (every sensation) under some unanalyzed complex" (pp. 19-20).

"Bodies do not produce sensations, but complexes of sensations (complexes of elements) make up bodies. If to the physicist bodies appear the real abiding existences, while sensations are regarded merely as their evanescent transitory show, the physicist forgets, in the assumption of such a view, that all bodies are but thought-symbols for complexes of sensations (complexes of elements)" (p. 22).

"For us, therefore, the world does not consist of mysterious entities, which by their interaction with another equally mysterious entity, the ego, produce sensations which alone are accessible. For us, colors, sounds, spaces, times, ... are the ultimate elements, whose given connection it is our business to investigate" (p. 23).

Mach arrived at his opinions through physics. James, whose opinions are essentially the same, arrived at them through psychology. In his *Psychology* they are not yet to be found, though there is a certain approach to them. The various articles containing the opinions which concern us at present are collected in the posthumous book called

Essays in Radical Empiricism. The following quotations will, I hope, serve to make it clear what these opinions are.

" 'Consciousness,' " says James, "is the name of a nonentity, and has no right to a place among first principles. Those who still cling to it are clinging to a mere echo, the faint rumor left behind by the disappearing 'soul' upon the air of philosophy. For twenty years past[2] I have mistrusted 'consciousness' as an entity; for seven or eight years past I have suggested its non-existence to my students, and tried to give them its pragmatic equivalent in realities of experience. It seems to me that the hour is ripe for it to be openly and universally discarded.

"To deny plumply that 'consciousness' exists seems so absurd on the face of it—for undeniably 'thoughts' do exist—that I fear some readers will follow me no farther. Let me then immediately explain that I mean only to deny that the word stands for an entity, but to insist most emphatically that it does stand for a function. There is, I mean, no aboriginal stuff or quality of being, contrasted with that of which material objects are made, out of which our thoughts of them are made; but there is a function of experience which thoughts perform, and for the performance of which this quality of being is involved. That function is *knowing*" (pp. 2-4).

"My thesis is that if we start with the supposition that there is only one primal stuff or material in the world, a stuff of which everything is composed, and if we call that stuff 'pure experience,' then knowing can easily be explained as a particular sort of relation towards one another into which portions of pure experience may enter. The relation itself is a part of pure experience; one of its 'terms' becomes the subject or bearer of the knowledge, the knower, the other becomes the object known" (p. 4).

After explaining the view, which he rejects, that ex-

[2] This article was first published in 1904.

perience contains an essential opposition of subject and object, he proceeds:

"Now my contention is exactly the reverse of this. *Experience, I believe, has no such inner duplicity: and the separation of it into consciousness and content comes, not by way of subtraction, but by way of addition*—the addition, to a given concrete piece of it, of other sets of experiences, in connection with which its use or function may be of two different kinds. The paint will also serve here as an illustration. In a pot in a paint-shop, along with other paints, it serves in its entirety as so much saleable matter. Spread on a canvas, with other paints around it, it represents, on the contrary, a feature in a picture and performs a spiritual function. Just so, I maintain, does a given undivided portion of experience, taken in one context of associates, play the part of a knower, of a state of mind, of 'consciousness'; while in a different context the same undivided bit of experience plays the part of a thing known, of an objective 'content.' In a word, in one group it figures as a thought, in another group as a thing. And, since it can figure in both groups simultaneously, we have every right to speak of it as subjective and objective both at once" (pp. 9-10; the italics are in the original).

"Consciousness connotes a kind of external relation, and does not denote a special stuff or way of being. *The peculiarity of our experiences, that they not only are, but are known, which their 'conscious' quality is invoked to explain, is better explained by their relations—these relations themselves being experiences—to one another*" (p. 25; the italics are in the original).

James explains, a few pages later, that a vivid image of fire or water is just as truly hot or wet as physical fire or water. The distinction, he says, lies in the fact that the imagined fire and water are not causally operative like the "real" fire and water. "Mental fire is what won't burn

real sticks; mental water is what won't necessarily (though of course it may) put out even a mental fire. Mental knives may be sharp, but they won't cut real wood" (p. 33).

"The central point of the pure-experience theory is that 'outer' and 'inner' are names for two groups into which we sort experiences according to the way in which they act upon their neighbors. Any one 'content,' such as *hard,* let us say, can be assigned to either group" (p. 139).

Finally he comes to the alleged introspective certainty of consciousness. But his introspective deliverance is not the usual one. In himself, he says, "the stream of thinking (which I recognize emphatically as a phenomenon) is only a careless name for what, when scrutinized, reveals itself to consist chiefly of the stream of my breathing. The 'I think' which Kant said must be able to accompany all my objects, is the 'I breathe' which actually does accompany them. There are other internal facts besides breathing.... and these increase the assets of 'consciousness' so far as the latter is subject to immediate perception; but breath, which was ever the original of 'spirit,' breath moving outwards, between the glottis and the nostrils, is, I am persuaded, the essence out of which philosophers have constructed the entity known to them as consciousness" (p. 37).

In order to understand James's theory, it is necessary to consider more in detail his account of 'knowing.' Mere seeing and hearing, and sensation generally, he does not call 'knowing.' In all the cases where those who hold a different theory would say we have *direct* knowledge, there is, in James's view, no knowledge at all, but merely the presence of the thing itself as one of the constituents of the mind which is mistakenly supposed to know the thing. Knowing, according to him, is an external relation between two bits of experience, consisting in the fact that one of them leads to the other by means of certain intermediaries.

The following illustration aptly introduces his account of knowing:

"Suppose me to be sitting here in my library at Cambridge, at ten minutes' walk from 'Memorial Hall,' and to be thinking truly of the latter object. My mind may have before it only the name, or it may have a clear image, or it may have a very dim image of the hall, but such intrinsic differences in the image make no difference in its cognitive function. Certain *extrinsic* phenomena, special experiences of conjunction, are what impart to the image, be it what it may, its knowing office.

"For instance, if you ask me what hall I mean by my image, and I can tell you nothing; or if I fail to point or lead you towards the Harvard Delta; or if, being led by you, I am uncertain whether the hall I see be what I had in mind or not; you would rightly deny that I had 'meant' that particular hall at all, even though my mental image might to some degree have resembled it. The resemblance would count in that case as coincidental merely, for all sorts of things of a kind resemble one another in this world without being held for that reason to take cognizance of one another.

"On the other hand, if I can lead you to the hall, and tell you of its history and present uses; if in its presence I feel my idea, however imperfect it may have been, to have led hither and to be now *terminated*; if the associates of the image and of the felt hall run parallel, so that each term of the one context corresponds serially, as I walk, with an answering term of the other; why then my soul was prophetic, and my idea must be, and by common consent would be, called cognizant of reality. That percept was what I *meant*....

"In this continuing and corroborating, taken in no transcendental sense, but denoting definitely felt transi-

tions, *lies all that the knowing of a percept by an idea can possibly contain or signify*" (pp. 54-56).

It will be observed that, according to the above account, he usually ceases to "know" Memorial Hall when he reaches it; he only "knows" it while he has ideas which lead or enable him to perceive it by taking suitable steps. It is, however, possible, apparently, to regard an experience as "knowing" itself in certain circumstances. In an enumeration of cases, James says:

"Either the knower and the known are:

"1. the self-same piece of experience taken twice over in different contexts: or they are

"2. two pieces of *actual* experience belonging to the same subject, with definite tracts of conjunctive transitional experience between them; or

"3. the known is a *possible* experience either of that subject or another, to which the said conjunctive transitions *would* lead, if sufficiently prolonged" (p. 53).

In a later illustration, he says:

"To call my present idea of my dog, for example, cognitive of the real dog means that, as the actual tissue of experience is constituted, the idea is capable of leading into a chain of other experiences on my part that go from next to next and terminate at last in vivid sense-perceptions of a jumping, barking, hairy body. Those *are* the real dog, the dog's full presence, for my common sense" (p. 198).

And again: "Should we ever reach absolutely terminal experiences, experiences in which we were all agreed, which were superseded by no revised continuations, these would not be *true*; they would be *real*, they would simply *be* ....Only such *other* things as led to these by satisfactory conjunctions would be 'true'" (p. 204).

Before proceeding to examine the substantial truth or falsehood of James's theory, we may observe that his use of the word "experience" is unfortunate, and points to the

lingering taint of an idealistic ancestry. This word is full of ambiguity; it inevitably suggests an experiencing subject; it hints at some common quality, "being experienced," in all the constituents of the world, whereas there is reason to believe that no such common quality is to be found. This word is abandoned by Professor Perry, whose chapters on "A realistic theory of mind" and "A realistic theory of knowledge"[3] give an admirable account of the Mach-James hypothesis. Nevertheless, even in his account, as in the whole doctrine, it seems possible to detect the unconscious influence of an idealistic habit of mind, persisting involuntarily after the opinions upon which it was based have been abandoned. But this can only be made clear by a detailed examination of the grounds for and against the whole theory of neutral monism.

In favor of the theory, we may observe, first and foremost, the very notable simplification which it introduces. That the things given in experience should be of two fundamentally different kinds, mental and physical, is far less satisfactory to our intellectual desires than that the dualism should be merely apparent and superficial. Occam's razor, *"entia non multiplicanda praeter necessitatem,"* which I should regard as the supreme methodological maxim in philosophizing, prescribes James's theory as preferable to dualism if it can possibly be made to account for the facts. Again, "matter," which in Descartes's time was supposed to be an obvious datum, has now, under the influence of scientific hypotheses, become a remote super-sensuous construction, connected, no doubt, with sense, but only through a long chain of intermediate inferences. What is immediately present in sense, though obviously in some way presupposed in physics, is studied rather in psychology than in physics. Thus we seem to have here, in sense, a neutral ground, a watershed, from which we may pass either to

[3] Chaps. XII and XIII of *Present Philosophical Tendencies.*

"matter" or to "mind," according to the nature of the problems we choose to raise.[4]

The ambiguous status of what is present in sense is illustrated by the difficulties surrounding the notion of "space." I do not intend now to attempt a solution of these difficulties; I wish only to make them felt, lest it should seem as though space afforded a clear distinction between the material and the mental. It is still sometimes thought that matter may be defined as "what is in space," but as soon as "space" is examined, it is found to be incredibly ambiguous, shifting and uncertain. Kant's *a priori* infinite given whole, which merely expresses our natural beliefs whenever the difficult disintegrations of analysis escape from our memories, has suffered a series of shattering blows from the most diverse quarters. The mathematicians have constructed a multiplicity of possible spaces, and have shown that many logical schemes would fit the empirical facts. Logic shows that space is not "*the* subject-matter of geometry," since an infinite number of subject-matters satisfy any given kind of geometry. Psychology disentangles the contributions of various senses to the construction of space, and reveals the all-embracing space of physics as the outcome of many empirically familiar correlations. Thus the space of actual experience is appropriated by psychology, the space of geometry is appropriated by logic, and the space of physics is left halting between them in the humbled garb of a working hypoth-

[4] The neutrality of sensation in orthodox philosophy may be illustrated by the following quotation from Professor Stout's *Manual of Psychology*, p. 133: "If we compare the color *red* as a quality of a material object with the color *red* as a quality of the corresponding sensation, we find that redness as immediately perceived is an attribute common to both. The difference lies in the different relations into which it enters in the two cases. As a quality of the thing, it is considered in relation to other qualities of the thing,—its shape, texture, flavor, odor, etc. As a psychical state, it is considered as a peculiar modification of the consciousness of the percipient, in relation to the general flow of his mental life." There seems in this passage an acceptance, as regards sensation, of the doctrines of neutral monism which Professor Stout would be far from adopting generally.

esis. It is not in "space," therefore, that we can find a criterion to distinguish the mental and the physical.

A large part of the argument in favor of neutral monism, as stated by its advocates, consists in a polemic against the view that we know the external world through the medium of "ideas," which are mental. I shall consider this view in the next part; for the present, I wish only to say that, as against this view, I am in agreement with neutral monism. I do not think that, when an object is known to me, there is in my mind something which may be called an "idea" of the object, the possession of which constitutes my knowledge of the object. But when this is granted, neutral monism by no means follows. On the contrary, it is just at this point that neutral monism finds itself in agreement with idealism in making an assumption which I believe to be wholly false. The assumption is that, *if anything is immediately present to me, that thing must be part of my mind.* The upholders of "ideas," since they believe in the duality of the mental and the physical, infer from this assumption that only ideas, not physical things, can be immediately present to me. Neutral monists, perceiving (rightly, as I think) that constituents of the physical world can be immediately present to me, infer that the mental and the physical are composed of the same "stuff," and are merely different arrangements of the same elements. But if the assumption is false, both these opposing theories may be false, as I believe they are.

Before attempting a refutation of neutral monism, we may still further narrow the issue. Non-cognitive mental facts—feeling, emotion, volition—offer *prima facia* difficulties to which James offers a *prima facie* answer. His answer might be discussed, and might prove tenable or untenable. But as we are concerned with the theory of *knowledge*, we will ignore the non-cognitive part of the problem, and consider only what is relevant to knowledge.

It is in this sphere that his theory is important to us, and in this sphere that we must make up our minds as to its truth or falsehood.

Apart from objections depending upon argument, there is an initial difficulty in the view that there is nothing cognitive in the mere presence of an object to the mind. If I see a particular patch of color, and then immediately shut my eyes, it is at least possible to suppose that the patch of color continues to exist while my eyes are shut; so far, James would agree. But while my eyes are open, the patch of color is one of the contents of my momentary experience, whereas when my eyes are shut it is not. The difference between being and not being one of the contents of my momentary experience, according to James, consists in experienced relations, chiefly causal, to other contents of my experience. It is here that I feel an insuperable difficulty. I cannot think that the difference between my seeing the patch of red, and the patch of red being there unseen, consists in the presence or absence of relations between the patch of red and other objects of the same kind. It seems to me possible to imagine a mind existing for only a fraction of a second, seeing the red, and ceasing to exist before having any other experience. But such a supposition ought, on James's theory, to be not merely improbable, but meaningless. According to him, things become parts of my experience in virtue of certain relations to each other; if there were not a system of interrelated things experienced by me, there could not be one thing experienced by me. To put the same point otherwise: it seems plain that, without reference to any other content of my experience, at the moment when I see the red I am acquainted with it in some way in which I was not acquainted with it before I saw it, and in which I shall not be acquainted with it when it ceases to be itself present in memory, however much I may be able to recall various facts which would enable me to

see it again if I chose. This acquaintance which I have with what is part of my momentary experience seems to deserve to be called cognitive, with a more indefeasible right than any connected ideas such as James describes in speaking of Memorial Hall.

I shall return to the above difficulty, which seems to me the main objection to neutral monism, when I come to consider how the contents of my momentary experience are to be distinguished from other things; in this connection, the difficulty will take a more general form, and will raise questions which can be better considered after various more detailed difficulties have been dealt with.

The first difficulty which seems to require an answer is as to the nature of *judgment* or *belief,* and more particularly of erroneous belief. Belief differs from sensation in regard to the nature of what is before the mind: if I believe, for example, "that to-day is Wednesday," not only no sensation, but no presentation of any kind, can give the same objective content as is involved in my belief. This fact, which is fairly obvious in the above instance, is obscured, I think, by the unconscious habit of dwelling upon existential beliefs. People are said to believe in God, or to disbelieve in Adam and Eve. But in such cases what is believed or disbelieved is that there is an entity answering to a certain description. This, which can be believed or disbelieved, is quite different from the actual entity (if any) which does answer to the description. Thus the matter of belief is, in all cases, different in kind from the matter of sensation or presentation, and error is in no way analogous to hallucination. A hallucination is a fact, not an error; what is erroneous is a judgment based upon it. But if I believe that to-day is Wednesday when in fact to-day is Tuesday, "that to-day is Wednesday" is not a fact. We cannot find anywhere in the physical world any entity corresponding to this belief. What idealists have

said about the creative activity of mind, about relations being due to our relating synthesis, and so on, seems to be true in the case of error; to me, at least, it is impossible to account for the occurrence of the false belief that to-day is Wednesday, except by invoking something not to be found in the physical world.

In *The New Realism*[5] there is an essay called "A realistic theory of truth and error," by W. P. Montague. It will serve to illustrate the argument if we examine what is said on error in the course of this essay.

"The true and the false," says Mr. Montague, "are respectively the real and the unreal, considered as objects of a possible belief or judgment" (p. 252).

There is nothing unusual in this definition, yet it suffers from a defect so simple and so fundamental that it is amazing how so many philosophers can have failed to see it. The defect is that there is no such thing as the unreal, and therefore, by the definition, there can be no such thing as the false; yet it is notorious that false beliefs do occur. It is possible, however, that Mr. Montague might maintain that there are unreal things as well as real ones, for with him "real" is definable. His definition is as follows:

"The real universe consists of the space-time system of existents, together with all that is presupposed by that system" (p. 255).

He proceeds at once to deduce his view of the unreal:

"And as every reality can be regarded as a true identity-complex or proposition, and as each proposition has one and only one contradictory, we may say that the remainder of the realm of subsistent objects [i. e., the unreal] must consist of the false propositions or unrealities, particular and universal, which contradict the true propositions comprising reality" (*ibid.*).

From the above it appears that, according to Mr. Mon-

[5] By the American Six Realists, New York and London, 1912.

tague, (1) every reality is a proposition; (2) false propositions subsist as well as true ones; (3) the unreal is the class of false propositions. We cannot now pursue these topics, which belong to logic. But for reasons which I have set forth elsewhere, it would appear (1) that *no* reality is a proposition, though some realities are beliefs, (2) that *true* propositions have a certain correspondence with complex facts, while false propositions have a different correspondence, (3) that the unreal is simply nothing, and is only identical with the class of false propositions in the same sense in which it is identical with the class of simoniacal unicorns, namely in the sense that both are null. It follows, if it is not otherwise obvious, that belief involves a different kind of relation to objects from any involved in sensation and presentation. The typical error to Mr. Montague, as to neutral monists generally, is the so-called "illusion of sense," which, as I shall try to show fully on another occasion, is no more illusory or erroneous than normal sensation. The kind of error with which we are all familiar in daily life, such as mistaking the day of the week, or thinking that America was discovered in 1066, is forced into the mould of "illusions of sense," at the expense of supposing the world to be full of such entities as "the discovery of America in 1066"—or in any year that the ignorance of schoolboys may suppose possible.

A further difficulty, not wholly unallied to the difficulty about error, concerns the thought of non-temporal entities, or the belief in facts that are independent of time. Whatever may be the right analysis of belief, it is plain that there are times at which I am believing that two and two are four, and other times at which I am not thinking of this fact. Now if we adopt the view that there is no specifically mental element in the universe, we shall have to hold that "$2 + 2 = 4$" is an entity which exists at those moments of time when some one is believing it, but not at

other moments. It is however very difficult to conceive of an abstract fact of this sort actually existing at certain times. No temporal particular is a constituent of this proposition; hence it seems impossible that, except through the intermediary of some extraneous temporal particular, it should acquire that special relation to certain moments which is involved in its being sometimes thought of and sometimes not. It is, of course, merely another form of the same difficulty that we shall be compelled, if we adopt neutral monism, to attribute causal efficacy to this abstract timeless fact at those moments when it is being believed. For these reasons, it seems almost inevitable to hold that my believing that $2 + 2 = 4$ involves a temporal particular not involved in the object of my belief. And the same argument, word for word, applies also to presentations when their objects are not temporal particulars.

An analogous problem arises in regard to memory. If I remember now something which happened an hour ago, the present event, namely my remembering, cannot be numerically identical with the event of an hour ago. If, then, my present experience involves nothing but the object experienced, the event which I am said to remember cannot itself be the object experienced when I remember. The object experienced must be something which might be called an "idea" of the past event. To this, however, there seem to be the same objections, if taken (as it would have to be) as applying to *all* memory, that there are to the doctrine that all contact with outside objects occurs through the medium of "ideas"—a doctrine against which neutral monism has arisen as a protest. If the past can never be directly experienced in memory, how, we must inquire, can it ever come to be known that the object now experienced in memory is at all similar to the past object? And if this cannot be known, the whole of our supposed knowledge of the past becomes illusory, while it becomes

impossible to account for the obvious difference between our knowledge as regards the past and our knowledge as regards the future.

An objection, possibly not unavoidable, applies to James's account of "processes of leading" as constituting knowledge. His definition of the sort of "leading" required is vague, and would include cases which obviously could not be called knowledge. Take, for example, the instance, quoted above, of James's knowledge of his dog, which consists in the fact that "the idea is capable of leading into a chain of other experiences on my part that go from next to next and terminate at last in vivid sense-perceptions of a jumping, barking, hairy body." Obviously a great deal is unexpressed in this account. The original idea must have somehow "intended" the jumping, barking, hairy body: some purpose or desire must be satisfied when the dog appears. Otherwise, an idea which had led to the dog by accident would equally be cognitive of the dog. It is in this way, I suppose, that James was led to the pragmatic theory of truth. Ideas have many effects, some intended, some unintended; they will be cognitive, according to James, when they have *intended* effects, when we have the feeling "yes, that is what I was thinking of." At this point, the need of a neutral theory of desire becomes very urgent; but we will not dwell on this difficulty. The purely cognitive aspect of James's view offers sufficient difficulties, and we will consider them only.

The relations of cause and effect, which James supposes to intervene between the antecedent knowledge of his dog and the dog's actual presence, will require some further definition; for unintended sequences of cause and effect, even if their final outcome were what is intended, could not be said to show that the original idea was cognitive. Suppose, for example, that I wish to be with my dog, and start towards the next street in hopes of finding him there;

but on the way I accidentally fall into a coal-cellar which he has also fallen into. Although I find him, it cannot be said that I knew where he was. And apart from this difficulty, the causal relation is an extremely obscure one. I do not believe the received notions on the subject of causality can possibly be defended; yet, apart from them, James's account of the cognitive relation becomes obscure. There is in James and in some of his followers a certain *naïveté* towards science, a certain uncritical acceptance of what may be called scientific common sense, which seems to me largely to destroy the value of their speculations on fundamental problems. The notion of "a *chain* of experiences that go from next to next," if introduced in the definition of cognition, seems to me to show an insufficiently critical attitude towards the notion of causality. But I am not at all sure that this is a *vital* objection to James's view: it is not unlikely that it could be avoided by a re-statement.

Another difficulty is that, in order to make his account of cognition fit all cases, he has to include *potential* processes of leading as well as actual ones. Of the three kinds of relation which, according to him, may subsist between knower and known, the third, we saw, is described as follows: "The known is a *possible* experience either of that subject or another, to which the said conjunctive transitions *would* lead, if sufficiently prolonged." It is true he says (p. 54): "Type 3 can always formally and hypothetically be reduced to type 2," and in type 2 both experiences are actual. But by the word "hypothetically" he re-introduces the very element of possibility which he is nominally excluding: *if* you did such-and-such things (which perhaps in fact you do not do), your idea *would* verify itself. But this is a wholly different thing from *actual* verification. And the truth of a possible or hypothetical verification involves, necessarily, considerations which must sweep away verification altogether as the *meaning* of truth. It

may be laid down generally that *possibility* always marks insufficient analysis: when analysis is completed, only the *actual* can be relevant, for the simple reason that there is only the actual, and that the merely possible is nothing.

The difficulties in the way of introducing precision into the account of James's "processes of leading" arise, if I am not mistaken, from his having omitted to notice that there must be a *logical* relation between what is believed in the earlier stages and what is experienced in the fulfilment. Let us revert to the instance of Memorial Hall. According to James, I should be said to "know" Memorial Hall if, for example, I know that it is reached by taking the first turning on the right and the second on the left and then going on for about 200 yards. Let us analyze this instance. In the case supposed, I know, or at least I believe truly, the following proposition: "Memorial Hall is the building which is reached by taking the first turning on the right and the second on the left, and then going on for 200 yards." For brevity, let us call this proposition *p*. The name "Memorial Hall," in this proposition, may be assumed to occur as a *description,* i. e., to mean "the building called 'Memorial Hall.'" It may occur as a proper name, i. e., as a name for an object directly present in experience; but in the case supposed, when it is being questioned whether I know Memorial Hall at all, it is more instructive to consider the occurrence of the name as a description. Thus *p* asserts that two descriptions apply to the same entity; it says nothing about this entity except that the two descriptions apply to it. A person may know *p* (for instance, by the help of a map) without ever having seen Memorial Hall, and without Memorial Hall having ever been directly present in his experience. But if I wish to discover whether the belief in *p* is true or not, two courses are open to me. I may either search for other propositions giving other descriptions of Memorial Hall,

such as that it comes at such and such a point on the map; or I may proceed to discover the actual entity satisfying one of the descriptions, and then ascertain whether it satisfies the other. The order, as between the two descriptions, is theoretically irrelevant; but it happens that one of the two descriptions, namely the one telling me the way, makes it easy to find the entity described. I may therefore take the first turning on the right and the second on the left and proceed for 200 yards, and then inquire the name of the building in front of me. If the answer is "Memorial Hall," the belief in *p* is verified. But it seems a misuse of terms to say that belief in *p*, when *p* is in fact true, constitutes knowledge of Memorial Hall. Belief in *p* is belief in a proposition of which Memorial Hall itself is not even a constituent; it may be entertained, on adequate grounds, by a person who has never experienced Memorial Hall; it may be rejected erroneously by a person who vividly remembers Memorial Hall. And when I actually see Memorial Hall, even if I do not know that that is its name, and even if I make no propositions about it, I must be said to know it in some sense more fundamental than any which can be constituted by the belief in true propositions describing it.

If what has been said is correct, certain points emerge as vital. First, that James and his followers, like many other philosophers, unduly assimilate belief to presentation, and thereby obscure the problem of error; secondly, that what they call knowledge of an object is really knowledge of a proposition in which the object itself does not occur, but is replaced by a *description* in terms of images or other constituents of actual present experience; thirdly, that what makes such a proposition true is the relations of the constituents of this actual proposition, relations which *may* be (but need not always be) established by the intermediary of the object described, but even then are not rela-

tions into which the actual object described enters as a term or constituent. Thus what James calls knowledge of objects is really knowledge of propositions in which the objects do not occur, but are replaced by descriptions; and the constituents of such propositions are contained in the present experience of the person who is believing them.

This brings us to the last objection which I have to urge against neutral monism, namely the question: How is the group of my present experiences distinguished from other things? Whatever may be meant by "my experience," it is undeniable that, at any given moment, some of the things in the world, but not all, are somehow collected together into a bundle consisting of what now lies within my immediate experience. The question I wish to consider is: Can neutral monism give a tenable account of the bond which unites the parts of this bundle, and the difference which marks them out from the rest of the things in the world?

This problem is incidentally discussed by Professor Perry in his *Present Philosophical Tendencies,* in the chapter called "A Realistic Theory of Mind." He emphasizes first the fact that the same thing may enter into two different people's experience, and that therefore one mind's objects are not necessarily cut off from the direct observation of another mind. So far, I should agree. But it does not follow, unless neutral monism is assumed (if then), that one man can directly know that a certain thing is part of another man's experience. A and B may both know a certain object O, but it does not follow that A knows that B knows O. Thus the fact that two minds may know the same object does not show that they are themselves accessible to each other's direct observation, unless they *are* simply the objects which constitute the contents of their experience. In that case, of course, they must be accessible to each other's direct observation. Pro-

fessor Perry regards a shrinking from this conclusion as a mere mistake, due to the fact that so many of our objects are internal bodily states which, for physical reasons, are hidden from other observers. I cannot think that he is right in this. Consider something in no way private: suppose I am thinking $3 + 3 = 6$. I can know directly that I am thinking this, but no other man can. Professor Perry says:

"If you are a psychologist, or an interpreter of dreams, I may 'tell' you what is in my mind. Now it is frequently assumed by the sophisticated that when I thus verbally reveal my mind you do not *directly* know it. You are supposed directly to know only my words. But I cannot understand such a supposition, unless it means simply that you know my mind only *after* and *through* hearing my words" (p. 290).

This passage appears to me to embody a logical error, namely a confusion of universals and particulars. The meanings of words, in so far as they are common to two people, are almost all universals. Perhaps the only exception is "now."[6] If I say "this," pointing to some visible object, what another man sees is not exactly the same as what I see, because he looks from a different place. Thus if he takes the word as designating the object which he sees, it has not the same meaning to him as to me. If he attempts to correct this, he will have to replace the immediate datum of his sight by a description, such as "the object which, from the point of view of my friend, corresponds with the object which I see." The words, therefore, in which I try to tell my experience will omit what is particular to it, and convey only what is universal. (I do not mean that it is *logically* impossible for two men to know the same particular, but only that practically it does not occur, owing to difference of point of view.) It may be

[6] Even this exception is open to doubt.

said, however, that this difficulty does not apply in the case of an abstract thought consisting wholly of universal or logical constituents. In that case, it is true, I can convey wholly the *object* of my thought; but even then, there is something which I cannot convey, namely that something which makes my thought a particular dated event. If I think, at a certain moment, that $3 + 3 = 6$, that is an event in time; if you think it at the same moment, that is a second event at the same time. There is thus something in my thought over and above the bare logical fact that $3 + 3 = 6$; and it is just this something which is partly incommunicable. When I tell you that I am thinking that $3 + 3 = 6$, I give you information even if you are not wholly ignorant of arithmetic. It is this further something, which makes the thought *my* thought, that we have to consider.

On this point, Professor Perry says:

"When I am thinking abstractions, the contents of my mind, namely the abstractions themselves, are such as you also may think. They are not possessed by me in any exclusive sense. And the fact that they are my contents means that they are somehow bound up with the history of my nervous system. The contents, and the linkage which makes them mine, are alike common objects, lying in the field of general observation and study" (p. 297).

The important sentence here is "the fact that they are my contents means that they are somehow bound up with my nervous system." The same idea is expressed elsewhere in the same chapter. "Elements become mental content," he says *"when reacted to in the specific manner characteristic of the central nervous system"* (p. 299, his italics). And again, more fully:

"A mind is a complex so organized as to act desideratively or interestedly. I mean here to indicate that character which distinguishes the living organism, having originally the instinct of self-preservation, and acquiring in

the course of its development a variety of special interests. I use the term *interest* primarily in its biological rather than its physiological sense. Certain natural processes act consistently in such wise as to isolate, protect, and renew themselves" (pp. 303-4).

But such an account of what makes a mind seems impossible to reconcile with obvious facts. In order to know that such and such a thing lies within my experience, it is not necessary to know anything about my nervous system: those who have never learned physiology, and are unaware that they possess nerves, are quite competent to know that this or that comes within their experience. It may be—I have no wish either to affirm or deny it—that the things which I experience have some relation to my nervous system which other things do not have; but if so, this must be a late scientific discovery, built up on masses of observation as to the connections of the object of consciousness with the nervous system and with the physical object. The distinction between things of which I am aware—for instance, between the things I see before my eyes and the things behind my back—is not a late, elaborate, scientific distinction, nor is it one depending upon the relations of these things to each other. So much, I thing, is clear to inspection; I do not know how to prove it, for I cannot think of anything more evident. But if so, then neutral monism cannot be true, for it is obliged to have recourse to extraneous considerations, such as the nervous system, in order to explain the difference between what I experience and what I do not experience, and this difference is too immediate for any explanation that neutral monism can give.

We may now sum up this long discussion, in the course of which it has been necessary to anticipate many topics to be treated more fully at a later stage. Neutral monism, we saw, maintains that there are not two sorts of entities,

mental and physical, but only two sorts of relations between entities, namely those belonging to what is called the mental order and those belonging to what is called the physical order. In favor of the theory, we may admit that what is experienced may itself be part of the physical world, and often is so; that the same thing may be experienced by different minds; that the old distinction of "mind" and "matter," besides ignoring the abstract facts that are neither mental nor physical, errs in regarding "matter," and the "space" in which matter is, as something obvious, given, and unambiguous, and is in hopeless doubt as to whether the facts of sensation are to be called physical or mental. In emphasizing all this, we must acknowledge that neutral monism has performed an important service to philosophy. Nevertheless, if I am not mistaken, there are problems which this theory cannot solve, and there are facts which it cannot account for. The theory has arisen chiefly as a protest against the view that external objects are known through the medium of subjective "ideas" or "images," not directly. But it shares with this view the doctrine that whatever I experience must be part of my mind; and when this doctrine is rejected, much of its plausibility ceases.

The first and chief objection against the theory is based on inspection. Between (say) a color seen and the same color not seen, there seems to be a difference not consisting in relations to other colors, or to other objects of experience, or to the nervous system, but in some way more immediate, more intimate, more intuitively evident. If neutral monism were true, a mind which had only one experience would be a logical impossibility, since a thing is only mental in virtue of its external relations; and correspondingly, it is difficult for this philosophy to define the respect in which the whole of my experience is different from the things that lie outside my experience.

A second difficulty is derived from *belief* or *judgment*, which James and his followers unduly assimilate to sensation and presentation, with fatal results as regards the theory of error. Error is defined as "belief in the unreal," which compels the admission that there actually are unreal things.

A third difficulty is that the thought of what is not in time, or a belief in a non-temporal fact, is an event in time with a definite date, which seems impossible unless it contains some constituent over and above the timeless thing thought of or believed. The same point arises in regard to memory; for if what is remembered actually exists in the remembering mind, its position in the time-series becomes ambiguous, and the essential pastness of the remembered object disappears.

A fourth difficulty arises in regard to the definition of knowledge offered by James, though here it is hard to say how far this definition is essential to neutral monism. James considers throughout rather knowledge of things than knowledge of truths, and he regards it as consisting in the presence of other things capable of leading to the thing which these other things are said to know. Immediate experience, which I should regard as the only real knowledge of things, he refuses to regard as knowledge at all; and it would seem that what he calls knowledge of a thing is really knowledge of a proposition of which the thing is not even a constituent.

In addition to the above difficulties, there is a fifth, more fatal, I think, than any of them, which is derived from considerations of "this" and "now" and "I." But this difficulty demands considerable discussion, and is therefore reserved for the next part.

For these reasons—some of which, it must be confessed, assume the results of future discussions— I conclude that neutral monism, though largely right in its

polemic against previous theories, cannot be regarded as able to deal with all the facts, and must be replaced by a theory in which the difference between what is experienced and what is not experienced by a given subject at a given moment is made simpler and more prominent than it can be in a theory which wholly denies the existence of specifically mental entities.

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THE PRINCIPLES OF MECHANICS WITH NEWTON

FROM 1666 TO 1679.

NEWTON'S achievements with respect to the principles of mechanics have been considered by Mach.[1] I have tried to supplement this account by a thorough study of what is known or can be concluded about the development of Newton's ideas on the principles of mechanics. For this purpose it was necessary, above all, to make use of the manuscripts of Newton prior to the publication (1687) of the *Principia*. Many of these manuscripts have been carefully preserved in the library of Cambridge University, and these I have examined, thereby bringing to light a fact of importance, showing the occurrence of the conception of mass with Newton at an early date. Of the other manuscripts of Newton, the most important of those in the possession of the Royal Society of London and Trinity College, Cambridge, and some others, have already been published, and the new views suggested thereby—such as that of the important place which the determination of the attraction of a sphere of gravitating matter or an external point holds in the series of Newton's discoveries—require exposition and discussion, especially as they are not referred to by Mach.

[1] *Mechanics*, pp. 187-245. Dühring (*Kritische Geschichte der allgemeinen Principien der Mechanik*, 3d ed., Leipsic, 1887, pp. 172-211) gave an account of Newton's achievements, but by not examining Newton's conception of mass he neglected the discussion of the most important of these achievements—at least from the point of view of the principles of mechanics.

Indeed, the more we learn of Newton's work prior to 1687 and of the work of Newton's contemporaries, the more do we seem to be forced to the opinion that the importance of Newton's work was mainly due to the great mathematical advance which he made by the invention of his "method of fluxions" and his application of it to the problems of dynamical astronomy. It is not, as I think I have shown,[2] by a feat of the imagination that Newton is distinguished from his contemporaries; and the conception of mass and the principle of reaction, though never so clearly stated before Newton, had been accepted and used by many before Newton. Newton, owing to his mathematical ability of precisely formulating the principles he used in his mathematical physics, gave these principles a fairly clear expression,—and an expression in which many people, even at the present day, are acquainted with them to the exclusion of all other views, all historical knowledge, and all criticism. This one-sidedness must be regarded as most unfortunate, especially when certain defects in the Newtonian formulations are so plain to modern eyes.

Historical investigations bring before us the efforts of Newton's predecessors and contemporaries, and give us a truer idea of what Newton accomplished. Also these investigations sometimes serve the purpose of establishing the independence of two discoverers. In the present case we shall see most clearly that Newton's first investigations on circular orbits could not have been influenced by the results which, no doubt, Huygens had already obtained.

The present paper deals with Newton's work on mechanics from 1666 to 1679.

I.

The attempt to represent the movements of the heavenly bodies by a simple mathematical process was, appar-

[2] *Monist*, July, 1913.

ently, first made by Eudoxus of Cnidus[3] in Asia Minor (408-355 B. C.). According to him, the earth was the center of the universe, and round it rotated with uniform velocity spheres carrying the stars, sun, and so on. The irregularities in the motions of the sun and moon which were soon found to exist were explained by Hipparchus of Rhodes (about 190-120 B.C.) by supposing that, though the sun and moon moved uniformly in their orbits, yet the center of their orbits was not the center of the earth. The great successor of Hipparchus was Claudius Ptolemy of Alexandria, who carried on the work of astronomical observation from about 127 to 150 A. D. He worked out an extremely elaborate scheme by which he represented the motions of the planets with considerable accuracy by means of a complex apparatus of epicycles.

For more than 1300 years the Ptolemaic system remained without serious challenge, and it was Nicolas Copernicus[4] (1473-1543) who observed that the celestial motions could be described far more simply if the sun were supposed to be the fixed center of the planetary system and the earth and the other planets to revolve round it. This great discovery was only communicated orally by Copernicus to his disciples, on account of possible theological difficulties, but shortly before his death his great work *De revolutionibus orbium caelestium* was published by Rheticus (1543) with a dedication by Copernicus to the pope, and with the remark that it is not necessary that astronomical hypotheses be either true or probable, but that they accomplish their object if they reconcile calculation with observation. The earth having been abandoned as the center of the universe, a further sacrifice had to be made: the principle of uniform motion in a circle had

[3] Cf. E. W. Maunder, *The Science of the Stars,* London and Edinburgh, 1912, pp. 20-31.

[4] Brewster, *Memoirs of the Life, Writings, and Discoveries of Sir Isaac Newton,* Edinburgh, 1855, Vol. I, pp. 253-258.

also to be given up. This came as a result of the more refined observations of the Dane Tycho Brahe[5] (1546-1601).

The observations of Tycho Brahe enabled his friend and pupil Johann Kepler[6] (1571-1630) to subject the planetary motions to a far more searching examination than had yet been attempted. Kepler first endeavored to represent the planetary orbits by the hypothesis of a uniform motion in circular orbits; but, in examining the orbit of Mars, he found the deviations from a circle too great to be owing to errors of observation. He therefore compared the observations with various other curves, and was led to the discovery that Mars revolves round the sun in an elliptical orbit in one of the foci of which the sun is placed. By means of the same observations, he found that the radius vector drawn from the sun to Mars describes equal areas in equal times. These two discoveries were extended to all the other planets of the system and were published at Prague in 1609 in his *Nova astronomia seu physica caelestia tradita commentariis de motibus stellae Martis*. In 1619 he published at Linz his *Harmonia mundi,* containing his third great discovery that the periodic times of any two planets in the system are to one another as the cubes of their distances from the sun.

Kepler also made speculations on the subject of gravitation, and these and other theories of gravitation—for example, of Descartes, Bouillaud, Borelli, Huygens, and Hooke—before Newton, are described by Rosenberger in the book cited below.[7]

II.

The impressive nature of many of the Newtonian discoveries in mechanics tends to hide what is, perhaps, the most important aspect of Newton's work. Newton put into

[5] *Ibid.,* pp. 258-263.

[6] *Ibid.,* pp. 263-270.

[7] Pp. 135-157. Cf. Brewster, *op. cit.,* pp. 282-288.

our hands the key which unlocks many of the secrets of astronomy, such as the motions and attractions of sun and planets, and most people—even among scientific people—allow their admiration of Newton's results in celestial mechanics completely to hide what ought to be their admiration of what Newton did for the principles of dynamics. As Dühring[8] says: "The achievements of Newton in our domain [the principles of mechanics] are too often only appreciated under the influence of the applications of mechanics to the planetary system, and thus, especially by the English, relatively overestimated. By this, not only has an injustice been done to Galileo and Huygens, but that popular kind of judgment has been adopted in which the measure of appreciation is gauged, not by the consideration of the advances in point of principle but by the completed fact of a notable application and the astonishment at gigantic dimensions and discoveries so surprising in their spontaneity (*Unmotivirtheit*)." Lagrange,[9] comparing Galileo's telescopic discoveries with his contributions to mechanics, says that his mechanical investigations "did not bring him, in his life-time, as much celebrity as those discoveries which he made about the system of the world, but they are to-day the most enduring and real part of the glory of this great man. The discoveries of Jupiter's satellites, of the phases of the moon, of the sun-spots, and so on, only needed telescopes and assiduity; but extraordinary genius was needed to disentangle the laws of nature from phenomena which are always going on under our eyes, but of which the explanation had always eluded the search of philosophers."[10] We are tempted to remark that, in Newton's case, only mathematical talent and the method of fluxions (or the differential and integral cal-

[8] *Op. cit.*, p. 172.

[9] *Méchanique analitique,* Paris, 1788, p. 159; cf. *Œuvres de Lagrange,* Vol. XI, pp. 237-238.

[10] On Galileo's work, cf. also Brewster, *op. cit.*, pp. 270-282.

culus) were needed to work out the paths of the planets under their mutual attractions; but extraordinary genius was needed to discover the fundamental principles of mechanics which were necessary and sufficient in order that the application of mathematics might completely solve the problems of mechanics. However, we must remember that it was Newton himself who discovered the method of fluxions; while, on the other hand, no reasonable person can seriously maintain that Newton's formulation of the principles was wholly clear and cannot be improved upon. It is more correct to say that he succeeded in giving what is, in essentials, a *sufficient* foundation for mechanics: the foundation is not wholly *necessary*, in view of the facts that there is much tautology and at least one entity—mass—is "defined" by an apparent definition which really involves a vicious circle.

We are going to try to ascertain the growth of Newton's ideas on the principles of mechanics. This is made difficult by three circumstances: (1) In all historical or critical accounts known to me of Newton's contributions to the principles, only Newton's *Principia* of 1687, and not his earlier manuscripts and other sources of information, have been analyzed; (2) in those publications where some account is given of Newton's earlier manuscripts, attention has been paid, almost to the exclusion of everything else, to the growth of his methods for dealing with astronomical problems; (3) the interest of both the contemporaries of Newton and their successors has been fixed, for the reasons stated above, on Newton's discoveries in celestial mechanics, and consequently we find—as in all branches of science until, comparatively speaking, recent years—that fundamental principles, especially when the deductions from them impress us by their grandeur, are often accepted uncritically. If science is compared to a tree—which, we must remember, grows both upwards and down-

wards—we may say that the average man of science does not care to busy himself with the roots when such a fine prospect can be attained by climbing. The life of a tree is in its roots; and science, if its roots are not cultivated, becomes worse than dead science—it becomes living pedantry.

I have made a thorough study of all the books which appeared to me to cast any light on the development of Newton's ideas on the principles of mechanics before 1687, and also of those manuscripts which Newton wrote before 1687 on mechanics, and which have either been printed or are preserved in the library of the University of Cambridge.

III.

I will first of all give a list of authorities consulted.

Stephen Peter Rigaud: *Historical Essay on the First Publication of Sir Isaac Newton's Principia,* Oxford, 1838.

J. Edleston: *Correspondence of Sir Isaac Newton and Professor Cotes, including Letters of Other Eminent Men, now first published from the originals in the Library of Trinity College, Cambridge; together with an Appendix containing other unpublished Letters and Papers by Newton; with Notes, Synoptical View of the Philosopher's Life, and a Variety of Details illustrative of his History,* London and Cambridge, 1850.

Sir David Brewster: *Memoirs of the Life, Writings, and Discoveries of Sir Isaac Newton,* 2 vols., Edinburgh, 1855. A second edition—apparently unaltered, even as to the mistakes—was published at Edinburgh in 1860.

W. W. Rouse Ball: *An Essay on Newton's "Principia,"* London and New York, 1893.

Ferdinand Rosenberger: *Isaac Newton und seine physikalischen Principien. Ein Hauptstück aus der Entwickelungsgeschichte der modernen Physik,* Leipsic, 1895.

In addition to these, there is the Portsmouth Collection of scientific papers belonging to Newton, which is now in the possession of the University of Cambridge, and has been examined and arranged in the University library. A descriptive catalogue of it was published at Cambridge in 1888 under the title: *A Catalogue of the Portmouth Collection of Books and Papers written by or belonging to Sir Isaac Newton, the Scientific Portion of which has been presented by the Earl of Portsmouth to the University of Cambridge.* This catalogue was drawn up by the syndicate appointed on November 6, 1872, and the Preface is signed by H. R. Luard, G. G. Stokes, J. C. Adams, and G. D. Liveing.

An indication of the nature of the extant manuscripts and correspondence of Newton on scientific subjects is given in the above-cited book by Rouse Ball.[11] Both Rigaud and Brewster printed part of Newton's correspondence, but Rouse Ball has printed the most complete sets of correspondence between Hooke and Newton and between Halley and Newton. For these letters, then, I shall refer to Rouse Ball's book alone.

Fifty-nine letters from and to Newton were published by Rigaud in the second volume of *Correspondence of Scientific Men of the Seventeenth Century, including Letters of Barrow, Flamsteed, Wallis, and Newton, printed from the Originals in the Collection of the Right Honourable the Earl of Macclesfield,*[12] and a table of contents and in-

[11] *Op. cit.*, pp. 2-5. Here I will remark that the above books will always, in what follows, be referred to by the names of their respective authors followed by *"op. cit."* The reference *"Portsmouth Catalogue"* is to the catalogue just mentioned.

[12] Two volumes, Oxford, 1841. The book was posthumous and was edited by Rigaud's son, Stephen Jordan Rigaud. The letters in question are on pp. 281-437 of Vol. II, and begin with the date 1669. These letters do not seem to contain anything of interest in connection with our present subject, which has not passed over into Brewster. Pp. 242-254 are interesting in connection with the respective merits of Gregory's and Newton's telescopes. The first volume seems to contain little of interest.

dex were added by A. De Morgan[13] in 1862.[14] Neither name nor date appeared on the supplement itself.

We must also mention A. De Morgan's[15] review of Brewster's *Memoirs*.[16] De Morgan clearly showed that Brewster had succumbed to hero-worship, and had consequently been unjust to Leibniz and Flamsteed. It cannot be too strongly emphasized that De Morgan, by his deep historical researches, high standard of conduct, and eminent sanity of judgment, has made contributions of the utmost value to our knowledge and estimation of the character and religious opinions of Newton.

IV.

For the biography of Newton (1642-1727) the chief authorities are De Morgan, to whose essays we here refer once for all, Edleston, and Brewster. To Edleston's book is prefixed an admirable "Synoptical View of Newton's Life." We shall here be concerned almost wholly with his work up to and including the publication of the *Principia* in 1687. From a scientific point of view Newton's further life is of far less interest.

According to Brewster, the date of Newton's quitting Cambridge is written by his own hand in his commonplace book as 1665; and thus Brewster concluded that he left Cambridge "sometime before the 8th of August, 1665,[17] when the college was 'dismissed' on account of the plague, and it was therefore in the autumn of that year, and not

[13] According to p. 414 of the *Memoir of Augustus De Morgan*, by his wife Sophia Elizabeth De Morgan (London, 1882).

[14] Rouse Ball, *op. cit.*, p. 4. Here, too, is some information about the Macclesfield Collection.

[15] The review is unsigned, but. cf. (Mrs.) S. E. De Morgan's *Memoir of Augustus De Morgan*, p. 261.

[16] *The North British Review*, Vol. XXXIII, 1855, pp. 307-338.

[17] See also Brewster, *op. cit.*, Vol. I, note on p. 30.

in that of 1666 [as Pemberton said], that the apple is said to have fallen from the tree at Woolsthorpe, and suggested to Newton the idea of gravity." On the other hand, Newton himself said[18] that in the beginning of the year 1666 he procured a glass prism "to try therewith the phenomena of colors"; and, after further particulars, added:[19] "Amidst these thoughts I was forced from Cambridge by the intervening plague."

After Newton's return to Cambridge on the disappearance of the plague, he was elected Minor Fellow on the first of October, 1667. He took his degree of Master of Arts in 1668. In 1669 Newton succeeded Isaac Barrow in the Lucasian professorship of mathematics.

With Rouse Ball,[20] we consider the origin and history of the *Principia* previous to its compilation and publication (1685-1687) under three heads, which deal

a. with Newton's investigations in 1666;
b. with his investigations in 1679;
c. with his investigations in 1684, of which the chief results are embodied in the *De motu* of 1685.

Newton alluded to the subject of his early investigations in a letter to Halley of June 20, 1686:

"That in one of my papers writ (I cannot say in what year, but I am sure some time before I had any correspondence with Mr. Oldenburg, and that's) above fifteen years ago, the proportion of the forces of the planets from the sun, reciprocally duplicate of their distances from him, is expressed, and the proportion of our gravity to the moon's *conatus recedendi a centro terrae* is calculated, though not accurately enough."[21]

On July 14, 1686, Newton wrote to Halley:[22]

".... but for the duplicate proportion I can affirm

[18] *Ibid.*, p. 380. [19] *Phil. Trans.*, Vol. VI, p. 3075.
[20] *Op. cit.*, p. 2. [21] *Ibid.*, p. 157. [22] *Ibid.*, p. 165.

that I gathered it from Kepler's theorem about twenty years ago."

In a memorandum written, perhaps, about 1714,[23] Newton wrote that he had said "in the Introduction to the book of Principles":

"I found the Method [of fluxions] by degrees in the years 1665 and 1666. In the beginning of the year 1665 I found the method of approximating Series and the Rule for reducing any dignity of any Binomial into such a series. The same year in May I found the method of tangents of Gregory and Slusius, and in November had the direct method of fluxions, and the next year in January had the theory of colors, and in May following I had entrance into the inverse method of fluxions. And the same year I began[24] to think of gravity extending to the orb of the moon, and having found out how to estimate the force with which [a] globe revolving within a sphere presses the surface of the sphere, from Kepler's rule of the periodical times of the planets being in a sesquialterate proportion of their distances from the centers of their orbs I deduced that the forces which keep the planets in their orbs must [be] reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the

[23] *Ibid.*, pp. 6-7. This manuscript is in Section I, Division XI, Number 41, of the Portsmouth Collection, among unarranged fragments of papers relating to the dispute with Leibniz about the invention of the "Method of Fluxions." After the quotation of this note in the *Portsmouth Catalogue* is the remark (p. xix): "Newton appears to have made one or two mistakes of date, and, probably for this reason, has drawn his pen through the entire passage." On the other side of the page of manuscript is a reference to Leibniz's death as having taken place. Leibniz died in 1716, so that Rouse Ball's above conjectured date would appear to be two years too early.

[24] "The question of gravitation," says Rouse Ball (*op. cit.*, p. 6), "was one of the problems of the time, hence it was natural that he should consider it. It is, however, probable that at this period his inquiries on the subject were of but a slight character, and would have passed almost unnoticed had they not proved the earliest steps to his later discoveries. His conclusions were not published, nor, as far as we know, are they contained in any manuscript now extant." The contributions of Kepler, Gilbert, Bouillaud, Borelli, and Hooke to the theory of gravitation before Newton are dealt with by Brewster, *op. cit.*, Vol. I, pp. 268-269, 282-288, and very much more fully by Rosenberger, *op. cit.*, pp. 135-157. Newton mentioned Borelli and Bullialdus (Bouillaud) in a letter to Halley (Rouse Ball, *op. cit.*, pp. 159-160).

force requisite to keep the moon in her orb with the force of gravity at the surface of the earth, and found them answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded mathematics and philosophy more than at any time since. What Mr. Huygens has published since about centrifugal forces I suppose he had before me. At length in the winter between the years 1676 and 1677 [probably this should be 1679 and 1680] I found the proposition that by a centrifugal force reciprocally as the square of the distance a planet must revolve in an ellipse about the center of the force placed in the lower umbilicus of the ellipse and with a radius drawn to that center describe areas proportional to the times. And in the winter between the years 1683 and 1684 [this should be the winter between 1684 and 1685] this proposition with the demonstration was entered in the Register Book of the Royal Society.

"By this method [of fluxions] I invented the demonstration of Kepler's proposition in the year 1679, and almost all the rest of the more difficult propositions of the book of Principles in the years 1684, 1685, and part of the year 1686."

This account is not quite consistent with the more generally known and credited accounts of Whiston and Pemberton,[25] in which Newton is said to have thrown aside his calculation because of the discrepancy with observation—for he supposed a degree of latitude on the surface of the earth to contain sixty miles—and only to have taken them up again when the more exact measurements of Picard (1670) showed that the discrepancy was not very great after all.

Whiston and Pemberton's account was partly the basis

[25] Rouse Ball, *op. cit.*, pp. 8-11. It is more convenient to refer to this reproduction of Whiston and Pemberton's accounts in the *Memoirs* (1749) and *View of Sir Isaac Newton's Philosophy* (1728) respectively.

of that given in Sir David Brewster's *Memoirs of Newton*:[26] "It was doubtless in the same remarkable year 1666, or perhaps in the autumn of 1665, that Newton's mind was first directed to the subject of gravity. He appears to have left Cambridge some time before the 8th of August 1665, when the college was dismissed on account of the plague, and it was therefore in the autumn of that year, and not in that of 1666, that the apple is said to have fallen from the tree at Woolsthorpe, and suggested to Newton the idea of gravity. When sitting alone in the garden and speculating on the power of gravity, it occurred to him that, as the same power by which the apple fell to the ground was not sensibly diminished at the greatest distance from the center of the earth to which we can reach, neither at the summits of the loftiest spires nor on the tops of the highest mountains, it might extend to the moon and retain her in her orbit in the same manner as it bends into a curve a stone or a cannon ball, when projected in a straight line from the surface of the earth. If the moon was thus kept in her orbit by gravitation to the earth, or, in other words, its attraction, it was equally probable, he thought, that the planets were kept in their orbits by gravitating towards the sun. Kepler had discovered the great law of the planetary motions, that the squares of their periodic times were as the cubes of their distances from the sun, and hence Newton drew the important conclusion that the force of gravity or attraction by which the planets were retained in their orbits varied as the square at the earth's surface. He was therefore led to compare it with the force exhibited in the actual motion of the moon, in a circular orbit; but, having assumed that the distance of the moon from the earth was equal to sixty of the earth's semidiameters, he found that the force by which the moon

[26] The passage referred to occurs in Vol. I, pp. 25-27 (2d ed., Vol. I, pp. 23-24); cf. pp. 290-291 (2d ed., Vol. I, pp. 253-254).

was drawn from its rectilineal path in a second of time was only 13.9 feet, whereas at the surface of the earth it was 16.1 in a second. This great discrepancy between his theory and what he then considered to be the fact, induced him to abandon the subject and pursue other studies with which he had been previously occupied."

To the last sentence is attached a note on the story of the apple, to which we will shortly refer below.

Pemberton mentioned 1666 as the year in which the first thoughts that gave rise to the *Principia* occurred to Newton. This date was adopted by Rigaud, denied, as we have seen, by Brewster, and accepted, apparently on the grounds of the above manuscript account of Newton's, by Rouse Ball. Rigaud[27] remarks: "From Dr. Hodge's *Loimologia,* we learn that the plague began first in Westminster, at the end of 1664; that in London it was most violent in the hotter months of 1665, and had so far abated in the following winter that the inhabitants returned to their homes in December. The Royal Society did not indeed resume their regular meetings till March, 1666; but the disease had then subsided, and some writers have therefore referred Newton's speculation to 1665. Pemberton, however, is certainly right in assigning his retirement into Lincolnshire to the following year. In the sixth volume of the *Philosophical Transactions* there is a paper [already referred to] in which he says....that he was forced from Cambridge by the plague [in 1666]. Further, Edleston[28] gives 1666 as the year in which the first idea of gravitation occurred to Newton.

I have devoted some space to what may appear the unimportant question as to whether Newton's first speculations on gravity took place in 1665 or 1666. The reason why the question is not unimportant arises from the fact that Newton had found, in essentials, his method of fluxions

[27] *Op. cit.,* note on p. 1.

[28] *Op. cit.,* p. xxi.

in 1665 and the beginning of 1666.[29] When we reflect that it was this method that formed the indispensable instrument for the discovery of the motion of a particle round a center of force, we see the importance of this. Newton was accustomed to the handling of problems involving infinitesimal or fluxional considerations; such considerations enabled him, since he stood at the new point of view in mechanics reached by Galileo, to calculate at once the relation afterwards published by Huygens between the force to the center of a circular orbit, the uniform velocity of a body describing this orbit and the radius of the orbit. Such considerations also showed—and this is a very important point—that there was now no great difficulty in extending calculations to orbs other than circular. Indeed, as Newton said in the postscript to his letter of June 20, 1686, to Halley, any mathematician could have concluded, from Huygens's discovery of the law of force in circular orbits, the manner of finding such laws even when the orbits were not circular.[30]

That Newton *did* take the inaccurate estimate is very probable.[31] After the feat of imagination involved in supposing that the power of gravity extends as high as the moon,[32] he seems to have started from Galileo's law of inertia, upon which he always seems—perhaps because of these questions—to have laid a great and logically unnecessary stress, thus showing that it may help our minds to have the proposition B emphasized even when we have already admitted the proposition A and we know that A implies B. As Whiston[33] says: "An inclination came into

[29] Brewster, *op. cit.*, Vol. I, p. 25.

[30] Rouse Ball, *op. cit.*, p. 160.

[31] Cf. *ibid.*, pp. 15-16.

[32] Cf. Mach, *op. cit.*, pp. 189-190, 513-514; Rouse Ball, *op. cit.*, pp. 9, 11. With regard to the origin of these speculations, the story of the apple, which Mach (*op. cit.*, p. 510) mentions with scorn, seems to have some foundation (cf. Rouse Ball, *op. cit.*, pp. 11-12). Cf. on this point Rosenberger, *op. cit.*, pp. 119-120; Rigaud, *op. cit.*, pp. 1-2; Brewster, *op. cit.*, Vol. I, note on p. 27. The story was first circulated by Voltaire.

[33] Rouse Ball, *op. cit.*, p. 8.

Sir Isaac's mind to try whether the same power did not keep the moon in her orbit, notwithstanding her projectile velocity and which he knew always tended to go along a straight line, the tangent of that orbit — which makes stones and all heavy bodies with us fall downward, and which we call gravity." Similarly Pemberton says:[34] "As he sat alone in a garden, he fell into a speculation on the power of gravity: that as this power is not found sensibly diminished at the remotest distance from the center of the earth to which we can rise, neither at the tops of the loftiest buildings, nor even on the summits of the highest mountains; it appeared to him reasonable to conclude that this power must extend much farther than was usually thought; why not as high as the moon, said he to himself, and, if so, her motion must be influenced by it; perhaps she is retained in her orbit thereby. However, though the power of gravity is not sensibly weakened in the little change of distance at which we can place ourselves from the center of the earth, yet it is very possible that, so high as the moon, this power may differ much in strength from what it is here. To make an estimate what might be the degree of this diminution, he considered with himself that if the moon be retained in her orbit by the force of gravity, no doubt the primary planets are carried round the sun by the like power."

As a first approximation, Newton supposed the moon to be a particle revolving in a circular orbit about the earth as center. In the investigation which followed, Newton seems to have proved independently Huygens's proposition about the centripetal acceleration in a central orbit, that[35]

$$\varphi = v^2/r.$$

It is not unlikely, as Rouse Ball[36] remarks, that New-

[34] *Ibid.*, pp. 9-10.

[35] Mach, *op. cit.*, p. 160; *Ostwald's Klassiker*, No. 138, pp. 41-46, 74-76.

[36] *Op. cit.*, p. 13.

ton's proof of this relation was that referred to in his letter to Halley of July 14, 1686, when mentioning the form in which he proposed to get the scholium to the fourth proposition of the first book of the *Principia*: "In turning over some old papers, I met with another demonstration of that proposition, which I have added at the end of this scholium."[37] This proof is, as will be seen, based on infinitesimal principles:

"In any circle suppose a polygon to be inscribed of any number of sides. And if a body, moved with a given velocity along the sides of the polygon, is reflected from the circle at the several angular points; the force with which at every reflection it strikes the circle will be as its velocity: and therefore the sum of the forces, in a given time, will be as the velocity and the number of reflections conjointly; that is (if the species of the polygon be given), as the length described in that given time, and increased or diminished in the ratio of the same length to the radius of the circle; that is, as the square of that length applied to the radius, and therefore if the polygon, by having its sides diminished *in infinitum,* coincides with the circle, as the square of the arc described in a given time applied to the radius. This is the centrifugal force with which the body impels the circle, and to which the contrary force, wherewith the circle continually repels the body towards the center, is equal."

I cannot see[38] any grounds for supposing that Newton, in these early investigations, drew the conclusion from Kepler's law of areas that this law implies a central acceleration and a uniform speed if the orbit were a circle. Indeed the law of areas was avowedly demonstrated by Newton at a much later date. It is not necessary to suppose that Newton had verified exactly the very natural conjec-

[37] *Ibid.,* p. 165.

[38] Cf. J. Cox, *Mechanics,* Cambridge, 1904 and 1909, pp. 90-91. On Newton's early investigations, see also pp. 85-86.

ture that, if a body describes a circle uniformly, the deflective force is to the center of the circle. We must notice here that Kepler's laws were stated for the sun and the planets. That Newton considered the laws to apply also to the case of the earth and moon may perhaps be considered to be an indication that Newton had already extended the conception of gravitation to all matter, conformably to the third rule of his *regulae philosophandi*:

"....If it universally appears, by experiments and astronomical observations, that all bodies about the earth gravitate towards the earth, and gravitate in proportion to the quantity of matter which they severally contain; that the moon likewise, according to the quantity of its matter, gravitates towards the earth; that, on the other hand, our sea gravitates towards the moon; and all the planets mutually one towards another; and the comets in like manner towards the sun; we must, in consequence of this rule, universally allow that all bodies whatsoever are endowed with a principle of mutual gravitation. For the argument from the appearances concludes with more force for the universal gravitation of all bodies than for their impenetrability; of which, among those in the celestial regions, we have no experiments, nor any manner of observation."

Now, Kepler's third law (1619) is that the cubes of the major axes of the elliptical paths of the planets round the sun vary as the squares of the periodic times. Assuming that the earth bears much the same relation to the moon as the sun does to a planet, and that the orbit of the moon is circular, Kepler's third law states that

$$r^3/T^2 = k,$$

where r is the radius of the orbit, T the periodic time, v so that

$$v = 2\pi r/T,$$

and k is a constant. Substituting for v and then T in the above equation for φ, we get

$$\varphi = 4\pi^2/kr^2,$$

so that the force varies inversely as the square of the distance.

Newton[39] then calculated the space x through which the moon (considered as a particle) was drawn towards the earth in one second. He knew the space x' through which a particle near the surface of the earth was drawn by gravity in one second; it is about 16.1 feet. If his calculation showed that the ratio of x to x' was inversely as the ratio of the square of the distance of the moon from the earth's center, it would follow that the force which kept the moon in her (circular) orbit was the same as gravity. Newton made the numerical calculations, "though," as he says, "not accurately enough." Most likely they were made in the same way as that given in the fourth proposition of the third book of the *Principia*. This was as follows: The mean distance of the moon may be taken as $60a$, where a is the radius of the earth; the time of the moon is 39,343 minutes. Hence, in one minute the moon periodically describes an arc $2\pi . 60a/39343$, and, therefore, is drawn towards the earth through a distance equal to the versine of this arc. If a be taken as 4000 miles, this distance is about sixteen English feet, and it would follow that in one second the moon would be drawn through a distance x equal to $16/(60)^2$ feet; thus the moon and a particle near the earth would be drawn in one second through spaces very nearly inversely proportional to the squares of their distances from the earth's center; and the approximation is so close that we might reasonably infer that the force which keeps the moon in her orbit is the same as gravity. This verification requires that the value of a,

[39] Cf. Rouse Ball, *op. cit.*, pp. 13-15; Cox, *op. cit.*, pp. 92-93.

the earth's radius, shall be known approximately. Now, Pemberton and Whiston agree that Newton assumed that a degree of latitude on the earth contains 60 miles (whereas, in fact, it contains about 69½ miles). This is equivalent to saying that Newton supposed that the radius of the earth was $10800/\pi$ miles, that is to say, rather more than 3,400 miles (whereas, in fact, it contains about 4000 miles). Hence his value of a was about one-eighth too small, and thus his calculations seemed to show that in one second the moon fell through only seven-eighths of the distance through which gravity alone would have pulled it. The calculation with his data gives $x = 13.9$ feet.

This discrepancy does not appear, judging by Newton's own account given above, to have shaken his faith in the view that gravity, diminishing inversely as the square of the distance, acted on the moon, but led him to infer that some other force acted as well; and Whiston adds that Newton believed the other cause to be "Cartesius's vortices," while Pemberton states that Newton's "computation did not answer expectation; whence he concluded that some other cause must at least join with the power of gravity on the moon." But yet the view that Newton was quite disappointed by his calculation used to be generally held.

As regards the calculation of the distance fallen through by the moon in a second of time, Newton's object was to determine whether the moon, being deflected from the rectilinear direction of its motion, was drawn down towards the earth by a quantity which, according to the law which he assumed, would be consistent with the space through which heavy bodies in the same time have been found to fall at the earth's surface. The moon's distance being taken in semi-diameters of our globe, the absolute space through which it moved in its orbit during a given

time was to be deduced from the magnitude of the earth. Now Newton took the common measure of sixty miles for a degree of latitude, which had been used by the old geographers and seamen. This was considerably too small, and consequently the distance by which the moon would under such circumstances have been drawn from the tangent of its orbit, came out less than the truth. Possibly the quantity which formed the foundation of his calculation had been the more impressed on his mind from its agreement with the result of the observations which Edward Wright had published in 1610. Wright was a Cambridge man—a fellow of Caius College—which may have made his work more familiar to the members of the University.[40] With regard to the measurement published by Norwood in 1636, Voltaire[41] remarked that the civil wars "had buried it in oblivion"; but in this respect Voltaire was certainly mistaken, as Rigaud[42] showed from the number of editions through which Norwood's work passed, and from contemporary evidence. The question as to how and when Newton became acquainted with Picard's measurement of 1671, and other questions, were very fully discussed by Rigaud.[43] However, the modern idea, first advocated by Adams, and based upon Newton's own manuscript note among the Portmouth papers, is that Newton did not lay great stress upon the close accuracy of his 1666 calcu-

[40] Rigaud, *op. cit.*, p. 3. On the measurements of Snell and Norwood, see Rouse Ball, *op. cit.*, pp. 15-16. It may be mentioned that Newton possessed a copy of Robert Wright's (formerly of Jesus College, Cambridge) book *Viaticum Nautarun, or the Sailor's Vade Mecum* (see *Portsmouth Catalogue*, p. 56). I have hitherto been unable to find a copy of this work; the one in the Portsmouth Collection has been returned to Lord Portsmouth.

[41] See below.

[42] *Op. ct.*, pp. 4-5.

....[43] *Op. cit.*, pp. 6-12; cf. Brewster, *op. cit.*, Vol. I, pp. 290-291; Rouse Ball, *op. cit.*, pp. 22-23; *Principia*, Props. iv and xix, of Book III. Hooke, in his letter to Newton referred to below, of Nov. 24, 1679, alluded to Picard's geodetical measurements, and probably this led Newton to use Picard's estimate in the revision of his calculation. It should be mentioned that Snell's fairly accurate measure was mentioned in a book by *Varenius*, of which Newton brought out an edition in 1672 (Rouse Ball, *op. cit.*, p. 16).

lation, nor, consequently, upon Picard's new measurement. The whole question of the determination of forces and orbits seems to have appeared to him much more of merely a rough approximation than it actually was until, in 1685, he discovered his theorem on the attraction of spherical bodies. This will be considered more closely in the next section but one; and we will close the present section by giving a translation of Voltaire's account, in *Elémens de la Philosophie de Neuton,*[44] of Newton's early work on the theory of gravitation. This account is of great historical interest, as it was chiefly through Voltaire that Newtonianism became popular in France.

"One day in the year 1666, Newton, having retired to the country and seeing the fruits of a tree fall, fell, according to what his niece, Mrs. Conduitt, has told me, into a deep meditation about the cause that thus attracts all bodies in a line which, if produced, would pass nearly through the center of the earth. What, he asked himself, is the force which cannot arise from these imaginary vortices that have been proved to be false?[45] It acts on all bodies in proportion to their masses and not to their surfaces; it would act on the fruit which has just fallen from this tree if the tree were raised 3000 fathoms (*toises*)[46] or even 10,000 fathoms. If that is so, this force must act towards the center of the earth in the place where the moon is: if this is so, this power —whatever it may be—may, consequently, be the same as that which makes the planets tend towards the sun and as that which makes the satellites of Jupiter gravitate to Jupiter. Now it is demonstrated by all the inductions deduced from Kepler's laws that all these secondary planets

[44] First published in 1738. I quote from the new edition published at London in 1741, pp. 289-291.

[45] Voltaire, in fact, "proved" at the very beginning of the account of Newton's theory of gravitation that the Cartesian vortices and plenum are impossible.

[46] A *toise* is 6.39459 feet.

gravitate the more towards the center of their orbits as they are nearer and the less so as they are farther off, that is to say, reciprocally as the squares of their distances.

"A body placed in the position of the moon which revolves around the earth and a body placed near the earth must, then, both fall towards the earth exactly according to this law. Thus, to be assured if it is the same cause which retains the planets in their orbits and which makes heavy bodies fall to the earth, only measurements are necessary; we have only to examine what space is described by a heavy body falling to the earth in a given time and what space would be described, in a given time, by a body placed in the region of the moon. The moon itself is this body, and it may be considered as falling from its highest point of the meridian....

"It was thus that Newton reasoned; but he used, for the measurement of the earth, the faulty estimate of sailors, who counted sixty English miles for a degree of latitude, whereas it should have been seventy miles.

"There was, indeed, a more correct measurement of the earth. Norwood, an English mathematician, had measured a degree of the meridian fairly exactly in 1636 and had found it to be about seventy miles. But Newton did not know of this measurement made thirty years before. The civil wars which had afflicted England, and which are as harmful to the sciences as to the state, had buried in oblivion the only correct measurement of the earth that there was; and the rough estimate of sailors was kept. By this calculation the moon was too close to the earth, and the proportions sought by Newton were not given exactly. He did not think that he might add anything to fit nature to his ideas; he wished to fit his ideas to nature. He abandoned, therefore, the beautiful discovery which analogy with the other stars made so probable and to which so little

was lacking for demonstration; and this rare quality of Newton's must give a great weight to his opinions.

"Finally, with the more exact measurements made in France, he found the demonstration of his theory. . . . Newton thus took up again the thread of his demonstration."

We must remember that this extremely naive and superficial account had—perhaps because of the enthusiasm with which it was written—an enormous influence on eighteenth century France. Indeed, its influence extended far beyond the boundaries of France; perhaps the conclusion—not wholly made legitimate, in this instance at least, by what appears really to have happened—as to Newton's sublime scientific restraint may not have been without influence on Cox's *Mechanics*.[47]

VI.

The first great mathematical advance of Newton's lay in the determination of the law of force for orbits which were not circular. The treatment of circular orbits is simple,[48] but in the more general case account has to be taken of a curvature which varies from point to point.

The conception of circle of curvature was introduced into geometry by Kepler in 1609,[49] and there seem to have been no other investigations on curvature published before 1665, when Newton applied his newly discovered fluxions to "the finding the radius of curvity of any curve." The remarks of Wallis,[50] which arose out of a controversy about the angle of contact of a circle with its tangent, are of later date than this, and were not published till 1685.

The normal to a given curve at a given point (P) will intersect with the normal at a neighboring point (Q) at

[47] Cambridge, 1904 and 1909.

[48] Cf. Mach, *Mechanics*, pp. 158-161.

[49] Moritz Cantor, *Vorlesungen über Geschichte der Mathematik*, Vol. II, 2d ed., Leipsic, 1900, p. 603.

[50] *Ibid.*, Vol. III, 2d ed., Leipsic, 1901, pp. 26-27; cf. Vol. II, p. 687.

some point (C') on the concave side of the curve. As Q approaches P, C' will—at least in the case of all the curves considered by Newton and by his successors until comparatively modern times—approach a limiting position C. The circle described with center C and radius CP will fit the curve more closely in the neighborhood of P than does any other circle. This circle is called "the circle of curvature at P." The "radius of curvature" (CP) is a measure of the curvature of the curve; the smaller the radius of curvature, the sharper is the turning of the curve. We find CP by finding the limit of $C'P$. Now

$$C'P/PQ = \sin PQC'/\sin PC'Q,$$

and $PC'Q$ is equal to the angle between the tangents at P and Q. Further, as Q approaches P, the angle PQC' approaches a right angle, and $PC'Q$ ultimately becomes nearly equal to its sine and then vanishes. Hence, in the usual notation, we find for CP the value $ds/d\varphi$.

Notice that we have to imagine, when we are dealing with a body describing a non-circular orbit, that, in the infinitely small interval of time from t to $t + dt$, the body describes a small circular arc of length ds and radius r, where $r = ds/d\varphi$, $d\varphi$ being the small angle subtended by the arc ds at the center of the circle. In general, r will vary from point to point; if the orbit is such that $ds/d\varphi$ is the same as s/φ, so that r is the same however large the arc s may be, the orbit is circular. We see, then, that we have only to apply simple infinitesimal considerations to pass from circular orbits, considered by Huygens and Newton independently, to the general case.

It may be added that the "curvature" at a point is usually defined to be $d\varphi/ds$ or $1/r$; and that the radius of curvature may be expressed in terms of the rectangular coordinates by means of the well-known formulae $ds^2 = dx^2 + dy^2$, and $\tan\varphi = dy/dx$, so that

$$d\varphi = [(d^2y/dx^2)dx]/[1 + (dy/dx)^2].$$

Now, let v be the tangential velocity of the body at P and $v + dv$ that at Q, and notice that $v = ds/dt$. The resolved part of the velocity at Q in a direction parallel to the normal at P is $(v + dv)\sin(d\varphi)$ or, neglecting the infinitesimal of the second order, $v.d\varphi$. The normal acceleration, then, at P is $(v.d\varphi)/dt$, or

$$v(d\varphi/ds)(ds/dt) = v^2/r,$$

where r is the radius of curvature at P. This is a generalization of the theorem of Huygens and Newton.

Another way of obtaining the same result is to call f the central acceleration, and to notice that the tangential velocity at Q is the result of compounding the tangential velocity v at P and the velocity $f.dt$ perpendicular to it. Now

$$(f.dt)/v = \tan(d\varphi) = d\varphi,$$

and, remembering that the angle PC′Q is equal to $d\varphi$,

$$d\varphi = (v.dt)/r;$$

so

$$f = v^2/r.$$

VII.

We now meet the question, referred to in the preceding section, which has done much towards the changing of our point of view towards the long delay and perhaps the lack of interest of Newton in the pursuit of the theory of gravitation.

In a letter to Halley of June 20, 1686, Newton wrote:[51] "....I never extended the duplicate proportion lower than to the superficies of the earth and, before a certain demonstration I found the last year, have suspected that it did not reach accurately enough down so low; and there-

[51] Rouse Ball, *op. cit.*, p. 157.

fore in the doctrine of projectiles I never used it nor considered the motions of the heavens;...."

The theorem in question contains the discovery that a spherical mass attracts an external particle as if the whole mass were concentrated at the center. We must remember that every particle of the mass is supposed to attract, so that this result is as simple as it is beautiful. Newton's papers and correspondence of 1679 and 1685 show, according to Rouse Ball,[52] that at this time he did not know how to determine the attraction of a solid body, like the sun or earth, on an external particle; he did not think that the earth's attraction was directed exactly to its center; and it is probable that, previously to 1685, he considered that the attraction of a planet on its moon (or of the sun on a planet) could be regarded as directed to a fixed point, and varying as the inverse square of the distance from it, only when the two bodies were at a distance so great that their dimensions could be neglected compared with the distance between them.

The extension of the idea of gravitation to the smallest particles of bodies is not only natural, but even seems to be, speaking psychologically, inevitable. Further, Newton's calculations on the moon's orbit were only very rough, and he could hardly have been deterred from following up the new idea by the merely approximate character of the result. These two considerations, combined with the probable cause mentioned above for the cessation of Newton's investigations for thirteen years, seem to have induced the celebrated English astronomer John Couch Adams[53] to believe that Pemberton and Whiston were mistaken as to the insufficiency of the verification. Newton knew that the orbit was not actually circular, and that his numerical data were only approximate; hence he could have expected only a rough verification of the hypothesis,

[52] *Ibid.*, p. 11.

[53] Rouse Ball, *op. cit.*, pp. 16-17.

and, as he asserted that he found his results agree or "answer pretty nearly," Adams considered that these calculations were sufficient to convince Newton that it was gravity alone that retained the moon in its orbit, and further, he strongly suspected that Newton already believed that gravity was due to the fact that every particle of matter attracts every other particle, and that this attraction varies as the product of the masses and inversely as the square of the distance between them. We will return to this point further on.

VIII.

It is important to bear in mind the fact that Newton could not have made use of the researches of Huygens on centrifugal force. Newton's first speculations and calculations on gravitation began in 1666, while Huygens's theorems on centrifugal force were given[54] in 1673, in the fifth part of the *Horologium oscillatorium*— but without proof—and a fairly complete "Tractatus de vi centrifuga" was only published in Huygens's *Opuscula Postuma* of 1703.[55]

A reader of Mach's *Mechanics*[56] might conclude that Newton followed in the footsteps of Huygens; but in point of fact it was Galileo's work alone which was the starting point for Newton's early reflections on gravitation and the moon's orbit.

[54] Some theorems were communicated by Huygens to the London Royal Society in 1669 in the form of an anagram.

[55] A convenient annotated German translation of this tract is given in No. 138 of *Ostwald's Klassiker*.

[56] P. 188. Mach here says: "He who clearly understands the doctrine of Galileo and Huygens, must see that a curvilinear motion implies deflective acceleration. Hence, to explain the phenomena of planetary motion, an acceleration must be supposed constantly directly towards the concave side of the planetary orbits." Of course this is not a statement that Newton made use of Huygens's discoveries, but it might be understood as implying that he did so. Dühring (*op. cit.*, p. 173) wrongly stated that Newton's theory partly grew out of Huygens's theory of motions in a circle under a central force. The fact that the beginnings of Newton's investigations of centripetal forces

"It is not merely," says Brewster,[57] "from his astronomical discoveries, brilliant as they are, that Galileo claims a high place in the history of Newton's discoveries. His profound researches on mechanical science—his determination of the law of acceleration in falling bodies—and his researches respecting the existence and cohesion of solid bodies, the motion of projectiles, and the center of gravity of solids, have ranked him among the most distinguished of our mechanical philosophers. The great step, however, which he made in mechanics was the discovery of the general laws of motion uniformly accelerated, which may be regarded as the basis of the theory of universal gravitation." Indeed, it seems that nobody can seriously maintain that Galileo's mechanical researches did not exert a far more important influence on Newton's work than did his astronomical discoveries. Undoubtedly, descriptions of the sun, moon, and planets are more striking to an ordinary person than a description of the law followed by falling bodies near the surface of the earth; but there can be no comparison between the essential importance to science of these descriptions. And we shall see that the correct mechanical principles to be followed in the formulation of the problem of finding the path of a particle which moves under the influence of a central attraction was familiar, for example, to Wren, Hooke, and Halley. These men were stopped by mathematical difficulties; the ideas of the fluxional or infinitesimal calculus were necessary for the progress of mechanics. It was fortunate that Newton had become familiar with the help that the conception of motion may offer in the solu-

were prior to the publication of Huygens's work was emphasized by J. Cox in his *Mechanics* (p. 90), a text-book written on the historical lines laid down so splendidly by Mach, and thus combining teaching, history, and criticism in a welcome manner. Cox (*op. cit.*, pp. 88-89) also laid stress on Galileo's law of inertia, which was and is so important psychologically to any one attacking problems of celestial mechanics.

[57] *Op. cit.*, Vol. I, pp. 273-274.

tion of certain geometrical problems and had invented the method of fluxions at a very early date.

IX.

With regard to the progress that Newton made between 1666 and 1679 in his ideas on gravitation, we have clear indications from Newton's own letters that he was familiar with the fact that gravitation varied inversely as the square of the distance by 1673 at the latest. When Huygens published his *Horologium oscillatorium* in 1673, he presented a copy to Newton, and Newton, in his letter of thanks, said:[58] "I am glad that we are to expect another discourse on the *Vis centrifuga,* which speculation may prove of good use in natural philosophy and astronomy, as well as mechanics. Thus, for instance, if the reason why the same side of the moon is ever towards the earth be the greater *conatus* of the other side to recede from it, it will follow (upon supposition of the earth's motion about the sun) that the greatest distance of the sun from the earth is to the greatest distance of the moon from the earth not more than as 10,000 to 56, and therefore the parallax of the sun is not less than 56/10,000 of the parallax of the moon; because, were the sun's distance less in proportion to that of the moon, she would have a greater *conatus* from the sun than from the earth. I thought also some time that the moon's liberation might depend upon her *conatus* from the sun and earth compared together till I apprehended a better cause."

The rest of this letter, which, for the most part, is on the subject of colors, was printed in the *Philosophical Transactions* of July 21, 1673. From the above words, "it is evident," continues Newton in his letter to Halley,

[58] The copy of this letter to Huygens was given in a letter of Newton to Halley of July 27, 1686 (Rouse Ball, *op. cit.*, p. 166). Before Newton found the copy of this letter, he gave from memory, in his letter to Halley of June 20, 1686, a slightly different account which is referred to below.

"that I was at the time versed in the theory of the force arising from circular motion, and had an eye upon the forces of the planets, knowing how to compare them by the proportions of their periodical revolutions and distances from the center they move about: an instance of which you have here in the comparison of the forces of the moon arising from her menstrual motion about the earth and annual about the sun."

The "better cause" mentioned in the above letter to Huygens was communicated in 1675 to Nicolas Mercator who published it in the following year in his *Institutiones astronomicae.* Galileo had discovered and explained the diurnal libration, arising from the spectator not viewing the moon from the center of the earth, but it was reserved for Newton to explain the libration in longitude which Hevelius, its discoverer, had ascribed to the displacement of the center of the moon's orbit from the center of motion. Newton showed that it was occasioned by the inequalities of the moon's motion in an elliptic orbit round the earth, combined with the uniformity of her motion round her axis. In the same letter to Mercator, he showed that the libration in latitude arose from her axis of rotation being inclined 88° 17′ to the ecliptic.

Before Newton had found the copy of his letter of 1673 to Huygens, he wrote to Halley on June 20, 1686, that he "gave those rules in the end [of Huygens's book] a particular commendation for their usefulness in philosophy, and added out of my aforesaid paper [an early manuscript, probably of 1666] an instance of their usefulness, in comparing the forces of the moon from the earth, and earth from the sun, in determining a problem about the moon's phase, and putting a limit to the sun's parallax; which shows that I had then my eye upon comparing the forces of the planets arising from their circular

motion, and understood it."[59] In a postscript to this letter, Newton added: "It is more than I can affirm that the duplicate proportion was not expressed in that letter."[60]

However, in the letter of July 27, 1686, Newton acknowledged: "Now, though I do not find the duplicate proportion expressed in this letter (as I hoped I might), yet, if you compare this passage of it here transcribed with that hypothesis of mine, registered by Mr. Oldenburg in your books, you will see that I then understood it. For I there suppose that the descending spirit acts upon bodies here on the superficies of the earth with force proportional to the superficies of their parts; which cannot be, unless the diminution of its velocity in acting upon the first parts of any body it meets with be recompensed by the increase of its density arising from that retardation. Whether this be true is not material. It suffices that it was the hypothesis. Now, if this spirit descend from above with uniform velocity, its density, and consequently its force, will be reciprocally proportional to the square of its distance from the center. But if it descend with accelerated motion, its density will everywhere diminish as much as its velocity increases; and so its force (according to the hypothesis) will be the same as before, that is still reciprocally as the square of its distance from the center."[61]

The hypothesis referred to in this letter was contained in the discourse presented in manuscript dated the 7th of December, 1675, to the Royal Society, with the title: "A Theory of Light and Colours, containing partly an Hypothesis to explain the properties of light discoursed of by him in his former papers, partly the principal phenomena of the various colours exhibited by thin plates or bubbles, esteemed to be of a more difficult consideration, yet to depend also on the said properties of light."[62]

[59] Rouse Ball, *op. cit,* pp. 157-158. [60] *Ibid.,* p. 160. [61] *Ibid.,* pp. 166-167.

[62] This paper was read on Dec. 9, 1675 (Birch, *op. cit.,* Vol. III, pp. 247, 260), and registered in the January or February following.

It is curious to notice that not long before this date Collins informed James Gregory that Newton and Barrow had "begun to think mathematical speculations at least dry, if not somewhat barren."[63] The hypothesis,[64] together with the controversy with Hooke to which it gave rise, belongs more particularly to the subject of physical optics; but the subject of gravity was occasionally referred to. An ether was assumed, and its heterogeneity seemed to be required by "the electric and magnetic effluvia, and the gravitating principle.[65] Again, there occurred the paragraph:[66] "And as the earth, so perhaps may the sun imbibe this spirit copiously to conserve his shining and keep the planets from receding further from him; and they that will may also suppose that this spirit affords or carries thither the solary fuel and material principle of light. And that the vast ethereal spaces between us and the stars are for a sufficient repository for this food of the sun and planets. But this of the constitution of ethereal natures by the by."

Newton quoted this paragraph in the postscript of the letter written to Halley on June 20, 1686, with the remark: "In these and the foregoing words you have the common cause of gravity towards the earth, sun, and all the planets, and that by this cause the planets are kept in their orbs about the sun. And this is all the philosophy Mr. Hooke pretends I had from his letters some years after, the duplicate proportion only excepted. The preceding words contain the cause of the phenomena of gravity, as we find it on the surface of the earth, without any regard to the various distances from the center. For at first I designed to write of nothing more. Afterwards, as my manuscript

[63] Brewster, *op. cit.*, p. 127.

[64] The "Hypothesis" was reprinted by Brewster on pp. 390-409; and an extract from Birch's *History* (Vol. III, pp. 249-251) in Rigaud (*op. cit.*, Appendix No. xx, pp. 68-70).

[65] Brewster, *op. cit.*, p. 392.

[66] *Ibid.*, p. 394.

shows, I interlined the words above cited relating to the heavens; and, in so short and transitory an interlined hint of things, the expression of the proportion may well be excused. But if you consider the nature of the hypothesis, you will find that gravity decreases upwards, and can be no other from the superficies of the planet than reciprocally duplicate of the distance from the center, but downwards that proportion does not hold. This was but an hypothesis, and so to be looked upon only as one of my guesses, which I did not rely on; but it sufficiently explains to you why, in considering the descent of a body down to the center, I used not the duplicate proportion. In the small ascent and descent of projectiles above the earth, the variation of gravity is so inconsiderable that mathematicians neglect it. Hence the vulgar hypothesis with them is uniform gravity. And why might not I, as a mathematician, use it frequently, without thinking on the philosophy of the heavens, or believing it to be philosophically true?"[67]

In the body of this letter, Newton said:[68] "Between ten and eleven years ago, there was an hypothesis of mine registered in your books, wherein I hinted a cause of gravity from the superficies of the planets (though for brevity's sake not there expressed) can be no other than reciprocally duplicate of the distance from the center. And I hope I shall not be urged to declare, in print, that I understood not the obvious mathematical conditions of my own hypothesis."

In a letter to Robert Boyle, which was written on February 28, 1679,[69] Newton said: "I shall set down one conjecture more which came into my mind now as I was writing this letter: it is about the cause of gravity. For this end I will suppose ether to consist of parts differing

[67] Rouse Ball, *op. cit.*, pp. 161-162.

[68] *Ibid.*, p. 158.

[69] This letter is reproduced by Brewster, *op. cit.*, pp. 409-419. The passage given is on pp. 418-419.

from one another in *subtlety,* by indefinite degrees; that in the pores of bodies there is less of the grosser ether in proportion to the finer than in the regions of the air; and that yet the grosser ether in the air affects the upper regions of the earth and the finer ether in the earth the lower regions of the air, in such a manner that from the top of the air to the surface of the earth, and again from the surface of the earth to the center thereof, the ether is insensibly finer and finer. Imagine, now, any body suspended in the air or lying on the earth; the ether, being by the hypothesis grosser in the pores which are in the upper parts of the body than in those which are in its lower parts, and that grosser ether being less apt to be lodged in those pores than the finer ether below, it will endeavor to get out and give way to the finer ether below, which cannot be, without the bodies descending to make room above for it to go out into."

In this letter to Boyle, Newton deduced, from the rarification of the ether between two bodies which approach one another, firstly, an endeavor of these bodies to recede from one another, and, secondly, their adherence to one another.[70]

On this passage Rosenberger[71] remarks: "This deduction of molecular attraction and repulsion shows clearly how far Newton then was from his system of primitive or elementary forces, which only differed by the law of the actions, of all matter."

It is to be noticed that Newton himself, although he rejected the undulatory theory of light because he did not think that this theory could explain the rectilinear propagation of light, considered the whole of space to be filled with an elastic medium which propagates vibrations in a manner analogous to that in which the air propagates vibrations of sound. The ether penetrates into the pores

[70] Brewster, *op. cit.*, pp. 412-413. [71] *Op. cit.*, p. 125.

of all material bodies, whose cohesion it brings about, it transmits gravitational action, and its irregular turbulence constitutes heat. Light, although it is not itself constituted by vibrations of this medium, but is composed of currents of corpuscles emitted by luminous bodies, freely traverses the ether. The ether has a density so small that it does not offer any sensible resistance to the heavenly bodies which move in it.[72]

Here may be mentioned two facts which, indirectly or directly, concern theories of ether with Newton and his predecessors.

Bullialdus derived the conclusion that the force of gravity varies inversely as the square of the distance by analogy with the diminution that occurs in the intensity of light; the opinions of Robison and Leibniz on the validity of this argument were given by Rigaud.[73] The hypotheses framed by Bullialdus, Huygens, Leibniz, John Bernoulli, and Newton, to devise some mechanical cause for the effects of gravitation were also noticed by Rigaud.[74]

X.

About 1677, Donne and Newton visited Wren at his lodgings, and Wren "discoursed of this problem of determining the planetary motions upon philosophical principles." This was stated in a letter from Newton to Halley of May 27, 1686,[75] and Newton added: "You are acquainted with Sir Christopher. Pray know where and whence he first learnt the decrease of the force in a duplicate ratio of the distance from the center."

[72] Cf. E. T. Whittaker, *A History of the Theories of Aether and Electricity from the Age of Descartes to the Close of the Nineteenth Century*, London and Dublin, 1910. Cf. also p. 534 of Mach's *Mechanics*.

[73] *Op. cit.*, p. 56.

[74] *Op. cit.*, pp. 59-63, and Appendix, No. xviii, pp. 65-66, No. xx, pp. 68-70, and No. xvii, pp. 62-64 (letter to Boyle).

[75] Rouse Ball, *op. cit.*, p. 156.

of continuity or discontinuity is a matter of conviction rather than of reason. The fact is, we believe one or the other to be true and that is the end of it. In such discussions where a postulate is quite beyond our powers of verification by experience, each contestant chooses his starting point, and this once chosen, his argument in favor of it is as sound as that of his opponents. And the truth of this assertion is evident, because from these diametrically opposite postulates, conclusions which are quite identical are deduced by equally logical arguments; witness the controversy between the atomists and the Cartesians which see-saws through all scientific theory and is as far from settlement to-day as it was centuries ago. This dualism, as it may be called, is not confined to science; it is but one phase of that larger dualism of philospphy and ethics which has come down to us from the Greek thinkers; the claims of two opposing schools of thought are always present to show us that, if the mind can develop a system from one set of postulates, the same results can be obtained from their contraries.

If then our conception of the nature of things is but a matter of personal conviction, what value is there in the opinion of Professor Lorentz, that, if we wish to obtain an insight into the mysteries and operations of nature, we must make hypotheses? If such hypotheses are not capable of proof then they must rest on what Bolingbroke so aptly calls our inward sentiment of knowledge. The experimentalist who described phenomena from his inward sentiment of what they should be, rather than from observations of what they are, would be classed as a nuisance. Such a one not only does not advance our knowledge but he actually retards it, since his work must be repeated before the truth can be known. So, too, the theorist who relies on hypotheses and not on facts and laws builds a structure which is not only temporary and false but must

be torn down. If we relegate to metaphysics, so large a part of what is commonly called physics, what is left to the science of physics? The answer probably is to be found in the saying of Lord Kelvin, that no scientific statement is understood until it is measured. If we grant this, then we must examine, in such a discussion as this, our methods of measurement.

The statement of a phenomenon invariably contains two terms, called qualitative and quantitative factors; the former expressing "what kind" and the latter, "how much." Thus, when we speak of a distance as ten centimeters, we mean that we are to consider a quantity ten of the quality, length. Now it is not really the province of science to seek for absolute knowledge of either qualities or quantities, and the attempt to do so is the excuse for hypothesis. What science is concerned with, is the relative knowledge, or comparison, of different quantities of any given quality and the reduction of complex qualities into combinations of simpler ones.

It thus becomes of prime importance to settle on the simplest and most fundamental qualities which may serve as a foundation for our system of measurement. Because of the fact that mechanical motions and mechanical forces are the most readily perceived by us and are most easily expressed in mathematical formulae, the fundamental units of quality are always selected from mechanical concepts. Of these, length, time, and mass, are found to be incapable of further simplification and have been adopted as the units of measurement. As these qualities must enter into all equations quantitatively, and as we can have no conception of an absolute quantity, we have by statute fixed upon certain arbitrary standards of quantity, such as the centimeter and foot for length, the gram and pound for mass, the second for time. From these prime standards all other quantities of a mechanical nature may be readily derived;

as, for example, velocity is the quotient of a given length by a time; momentum is the product of a mass and its velocity; energy, one-half the product of a mass and a square of its velocity, etc.

The science of mechanics is the only branch of physics which has a completely developed theory. Not only is this the case, but the other branches of physics have been developed from a mechanical basis, in so far at least as the measurement of all phenomena is now made in terms of these mechanical units. This coordination in physics has the great advantage of making it possible to express what are apparently unrelated phenomena by combinations of only three standards of measurement and by a few general equations of mechanics, and we attain the additional advantage of expressing all these phenomena in the most concrete form imaginable. But, on the other hand, this method produces a serious break between physics and certain other sciences, especially those which rely more exclusively on the sense-perceptions as criteria for classification of phenomena. Thus, to the physicist light of different kinds is distinguished only by the three qualities possible in a mechanical wave; namely, its wave-length, the amplitude of its disturbance, and the complexity of its form. These same qualities must also serve to distinguish sounds, water waves and many other types of this kind of motion. For the biologist and psychologist, light is distinguished by its tint, intensity, and saturation, the three qualities which affect the sense of sight; and sound is determined by pitch, loudness, and timbre, which affect the auditory nerve. As there is no relation between the sensations of sight and hearing, so there can be no connection between light and sound. While the method of the physicist has the great advantage of unity, yet it suffers from its artificiality, as it tends to bring into undue prominence the mechanical energy involved in producing light

and sound and ignores the more important properties of these phenomena, their effect on our senses.

It is generally conceded that we have no abstract or absolute knowledge of a quantity of length, time, or mass. In other words, we can express any of these three quantities only as a numerical ratio with respect to a predetermined standard of the same quantity, such as a yardstick for length, a pound for mass, and a given motion of a clock-hand or of the rotation of the earth for time; and from this fact, it is often argued that we cannot form any idea of the qualities, space, time, and matter, except as they are individually and concretely measured. Kant, to be sure, maintained that we were endowed with an innate and inexplicable, but sufficient, idea of pure space and time. These qualities are, however, by themselves inappreciable to our senses. To make them sensible, we need a third, which he calls the *Ding an sich,* corresponding in the external world to what we call the entity, matter.

However vulnerable and unsatisfactory the doctrine of innate ideas may be, yet it seems to me that by the abstractive method as used in the science of mechanics we obtain a real and adequate idea of these three fundamental postulates. If we have not such a power of abstracting real ideas from our concrete observations of phenomena, then it is difficult to believe that the conclusions of Euclidean geometry and of mathematical analysis in general are rigorous. Thus, in mathematical definitions, space is a mere volume enclosed in an imagined boundary which may be conceived as of any extent from the indefinitely small to the indefinitely large. And this imagined bounding surface bears no closer relation to a concrete material envelope than does an image in a mirror to its object. It seems to me that I have an adequate idea, for instance, of the space in an empty room and that I can abstract all the properties from my concrete perception of the ma-

terial walls of this room except the one fact that they enclose this space. By deduction and by experience I am convinced that this space is not empty, but contains air. But it is really more difficult to appreciate the existence of the air, if it be at rest, than its non-existence. This belief is borne out by the slow advance in the conception of the existence of gases. And it also seems true to me that I have an adequate idea of an area enclosed in a triangle, because I can think of the area enclosed by three abstract lines and refrain from thinking of the concrete volume of any real lines drawn with a pencil.

As for the concept time, we have a twofold sense of it; one is the coincidence of an event with the position of the hands of a clock or of the earth. But we have, in addition, a much more general idea of time, as mere succession of events without any reference to such a standard of measurement as the second or minute. This might be called our belief that events entirely unconnected with our own experience occur successively just as those events do which we compare with a clock. We have, on awaking from sleep, a distinct and clear idea that events have transpired successively during our unconsciousness and without any reference to a measured interval of time. This general knowledge of "before and after" is apparently possible to an animal; a dog will with certainty expect a reward after the performance of a trick and not before or during it.

As for the third postulate, which is called matter, it may be taken in the most general sense as the something which makes space and time concrete to us. The necessity in science for such a postulate is clear, for science deals with the phenomena of an objective world. Even those men of science most opposed to the idea that mass is the measure of matter, still postulate an entity identical with matter, as just defined, although they may call it by a

different name, such as energy, electricity, etc. The distinction between mathematics and science lies in the conception of this third postulate. In pure mathematics matter becomes the abstract postulate, quantity. When discussing velocity abstractly there is then no need to consider the properties of the things moving, but in physics there is such a need. Or again, when dealing with space relations the mathematician is in no way limited to the restricted number of three dimensions which our sense-perception of the material universe imposes on science.

The supreme value of mathematics to science is due to the fact that scientific laws and theories have their best, if not their only complete, expression in mathematical formulae; and the degree of accuracy with which we can express scientific theory in mathematical terms is the measure of the state of a science. Thus it is possible to classify sciences according to their development, from the accumulation of statistics of phenomena to the generalization of these phenomena in comprehensive and rigorous laws. In such a classification, sociology or the study of existing society occupies the lowest rank, since true laws can be derived only from actions whose completed consequences are known. Sociology therefore attempts to found its laws on the data of history, the study of past society; history must in the same way rely on psychology, which deals with the actions of the individuals of society; psychology relies on biology; biology, on chemistry; chemistry, on physics; and physics on pure mathematics. While each science thus strives to found its laws on the conclusions of the following science, each succeeds only partially; this leads us to a paradox. The goal of science is mathematics, and while mathematics may be said to be the only true science since it has the only true scientific method, mathematics is no science because it deals with abstractions and ignores concrete phenomena.

As stated before, all quantities in mechanics may be expressed by combining the fundamental units—length, mass, and time—in simple ratios and products. And while the phenomena of light, heat, and electricity do not manifest themselves directly to our senses in any manner which may be coordinated with mechanical actions, yet we endeavor to measure them in physics as if they were mechanical in nature. Thus we speak of the velocity of light from the sun to the earth, as if we were considering a real mechanical motion of a ponderable body, although in the intervening space there is no matter. The only thing we have been able to observe is that light emitted from the sun appears on the earth some time later. Light as a phenomenon does not exist unless it is associated with matter; we can no more discuss the amount of light or its velocity in absolutely vacuous space than we can speak of the temperature of such space. Light has a true mechanical velocity when passing through space occupied by matter, for then we can observe and measure its path as well as the time function and so obtain a value for velocity. From observations on the velocity of light in matter and from analogy to the phenomena of sound transmission, which takes place only in spaces occupied by matter, we by the hypothetical method transfer the measurements and laws of light in transparent bodies to space not occupied by matter. The very assumption of a light-velocity in immaterial spaces requires us also tacitly to assume that *something,* in a mechanical sense, is moving. Once we have granted that light is something moving, then all the other phenomena of light permit of a hypothetical mechanical explanation, and we have the right to speak of the momentum and energy of this light something, whether it be corpuscles or waves. But it is the easiest thing in the world to forget that we can never obtain any real knowledge of this something we have called light. It is surprising that men of

science should believe that they have proved the existence of the ether, as when Sir Oliver Lodge states that it is the most massive thing imaginable; or that space is occupied by energy, an entity possessing inertia and probably gravitational force as Professor Einstein announces: they should see that such statements are not deductions made from our experimental knowledge of light, but are already contained in the hypothetical postulate that light is mechanical and has a mechanical velocity.

It is instructive to consider, in this connection, how we also have attached hypothetically the science of electricity to mechanics. The fundamental phenomenon observed, when bodies are electrified or magnetized, is that they attract or repel each other with a mechanical force which, like the force of gravitation, varies inversely as the square of the distance between them. In our fundamental units this mechanical force is equal to a mass times a length and divided by the square of a time. Now Coulomb, who discovered the law of electrical attraction, believed that electricity was a kind of fluid substance, such as was always introduced when phenomena were obscure; and with this idea in his mind, he employed the term "quantity" of electricity to indicate an analogy with a quantity of matter. On this supposition, a quantity of electricity expressed in mechanical units is equal to the square root of a length times a mass. Now it is quite certain that a quantity of electricity has nothing in common with length. Taking a step further, we find that the resistance of a conductor to an electrical current must be expressed as a velocity. Yet it would be absurd to attach any concrete relation between electrical resistance and mechanical velocity.

These three examples of velocity illustrate quite clearly the difference between the abstractive and hypothetical methods. From the definite and clear idea of the mechan-

ical velocity of a concrete body, we pass to an indefinite idea of the velocity of light in empty space, and in order to link this idea to mechanical notions, we assign to light a hypothetical material existence. But our mathematical equations lead us a step further and we can derive a purely formular relation between mechanical velocity and electrical resistance; in this case analogy between physics and mathematics entirely fails and no idea, even hypothetical, has been attached to the result.

The principles of the science of mechanics, on which the theories of the other branches have been built, date from the time of Galileo and Newton. As they had withstood the searching criticism of the masters of the science for centuries, without having been shown to be either false or inadequate, the belief grew that however other theories of physics might change, the laws of mechanics as stated by Newton were probably final. But it is inevitable that if we found the laws of light and electricity on mechanics, the time will come when the accumulation of knowledge will increase the discrepancies which must always exist between any two branches of science and will eventually require a thorough revision of one or the other. If the attention be directed more toward discovering the phenomena and laws of light and electricity than of mechanics, as it is to-day, any discrepancies will probably be laid to the laws of mechanics and their revision will be attempted to ensure agreement. This has occurred in the last few years, and the mechanics based on material bodies is being replaced by a mechanics of electricity or by one of energy, if the terms be permissible.

The mechanics of material bodies, to which the name of Newton is generally attached, was based on the objective reality of matter, whose quantitative measure was inertia or mass. Newton evidently considered inertia as a fundamental attribute of matter, and thus invariable and in-

explicable; something to be accepted and determined solely by experiment. Thus he says in his *Principia*: "Haec (materiae vis) semper proportionalis est suo corpore, neque differt quicquam ab inertia massae, nisi in modo concipiendi. Per inertiam materiae fit, ut corpus omne de statu suo vel quiescendi vel movendi difficulter deterbetur." This postulate may be freely translated to mean that the force of attraction of matter is always proportional to the amount of matter acting, and does not differ in any way from the inertia of mass except in our method of apprehending it. By inertia of matter is meant, that a body can be changed from its previous condition of rest or motion only by this material force. It is evident that he regarded inertia as an inherent and inalienable property of a body, independent of the influence of any other body or ether, and forming a connecting and stable link between ourselves and the external world. Such being the case, how may we decide what is the mass of any particular body? If a number of individuals measure experimentally a mass, or even if one of them measures it several times, no two observations will agree. Which observation gives the correct value? Newton would have answered, none of them. Data of objective phenomena can never be known exactly; each value we obtain approximates to the truth, and the approximation is the closer, the greater the number so obtained and the greater the care exercised. The final result must be deduced from all the observations, according to a well-developed mathematical theory of errors. The same reasoning was held to apply to observations on the space-dimensions of a body and on the time occurring during any event.

While the idea was advanced in this mechanics that the position and motion of any body could be determined only from the position of some other body, supposed for the time being to be at rest, or that position and motion

were relative and not absolute; yet it was not explicitly stated that there could not be absolute rest or motion, such as would occur if the motion of a body were referred to an absolutely fixed center of the universe or to an ether which was incapable of motion. For all practical problems, Newton's third law of motion, which states that to every action there is an equal and oppositely directed reaction, announces the universality of relativity.

We should finally note that mass, dimensions, and time were held to be unaffected by the motion of a body. Newton expresses this by saying that force-actions of matter, or the science of dynamics, are independent of its initial state of rest or motion.

The first serious criticism of these postulates was made by a number of physicists, forming what is often called the school of energetics, who proposed to substitute energy for mass as the fundamental attribute of matter. At the time, the change was rather immaterial as we were accustomed to think that mass and energy were coexistent and that either one was unintelligible without the other. It was the same kind of a problem as deciding which came first, the owl or the egg; the answer to which is that we know nothing about the matter.

The discrepancies in mechanics did not prove to be embarrassing until certain problems connected with light and electricity became pressing. Of these, three stand out most prominently.

The phenomena associated with electricity, when it passes through very high vacua, and with radioactive bodies like radium, are now explained as being due to the action of excessively small particles, carrying a charge of electricity and moving with a velocity comparable to the velocity of light, or about one hundred thousand miles per second. These particles, both because of their smallness and because of their velocity, are in an entirely different class

from the bodies previously considered in mechanics which have a sensible mass and whose greatest velocity is less than one hundred miles per second. Errors in mechanical laws which would otherwise be inappreciable may easily assume large proportions when applied to such extreme cases. The hypothesis of the electron has profoundly modified the conception of the atom, and the nature of matter and motion.

In the second place, recent experiments lead to the belief that matter is always electrified; that light is due to electrical variations; and that the amount of electricity associated with a given amount of matter is a fixed and invariable quantity. It was further shown by theory and confirmed by experiment that if an electrified body be moved, the ratio of its electrical charge to its mass must vary with the velocity of its motion; and since the electrical charge is held to be a constant, there remains only the alternative of considering the mass of a body as a variable. This evidently strikes at the very root of Newtonian mechanics. The effect of motion on mass is found to be inappreciable until the velocity approximates to that of light, so the discussion would have remained a purely academic one if the discovery of the electron had not brought us suddenly face to face with bodies which are supposed to have a velocity great enough to affect experimentally their mass. A further consequence of this theory is that mass becomes infinite when the velocity of light is reached. While this velocity has always been considered enormous and beyond our power to attain, such a result was entirely unsuspected. The laws of dynamics of bodies at rest could evidently no longer be held to be the same as those for bodies in motion.

Lastly, various experiments have been made to find an effect due to the mutual relations of the ether and matter. None could be found. One in particular, devised by

Professors Michelson and Morley, has proved to be the hardest problem in modern physics to explain. Calculation showed that certain properties of light should be affected appreciably by the motion of the earth through space, but the experiment proved beyond doubt that such was not the case. The first attempt at an explanation was made by assigning certain complicated motions to the ether which would annul the effect the motion of the earth should produce; but it was shown that unless the ether remained absolutely at rest worse difficulties resulted. Then the bold assumption was made, often called the Fitzgerald-Lorentz effect, that the dimensions of bodies in motion were so changed by their motion as to neutralize the effect of the earth's motion on the phenomena of light. Every body would, according to this idea grow shorter, the faster it moved; and would flatten out to nothing if it could attain a speed equal to the velocity of light.

One of the results of this criticism of mechanics has been to change profoundly our ideas of the ether. As is well known, the early conception of the ether was a kind of material substance possessing properties incompatible with those of any other kind of matter. A large part of the effort of theorists down to the middle of the nineteenth century was devoted to inventing attributes for the ether which would enable it to fulfil its function as the medium for the transmission of light. At this time, Faraday discovered an effect in electricity which required an entire change in our ideas of the ether. When he found that static electric charges and forces were dependent on the characteristics of ether, and when later Hertz showed experimentally that electromagnetic energy passed through the ether, and that this form of radiant energy was undoubtedly of the same type as light and heat; then it was found that no material ether could be imagined which would perform these new duties, especially as it was al-

ready staggering under the burdens of the old ones. Faraday proposed as a substitute that we should no longer imagine the ether to be a substance having material or mechanical properties, but one responding to electrical and magnetic actions. He was far in advance of his time and to be acceptable the hypothesis needed the mathematical development which was so skilfully accomplished by Maxwell. Both Faraday and Maxwell, while really destroying the material nature of the ether, strove to maintain at least a partial connection between electro-magnetic and mechanical attributes. For this reason they supposed that electro-magnetic stresses manifested themselves by creating mechanical strains in the ether and in matter immersed in it. Such strains must produce actual physical deformations of size and shape in all electrified bodies. It has been shown experimentally by the writer that such deformations are not produced in electrified matter and lately Professor Lorentz has stated, that since Helmholtz proved these stresses would cause the ether to move, and since no experiment has ever shown us a trace of a motion in the ether, we must deny the real existence of ether stresses. The effect of such a denial is to separate all attributes of the ether from concrete realities and to class them as abstract symbols. Professor Lorentz is quite explicit on this point, as he holds that, "while thus denying the real existence of ether stresses, we can still avail ourselves of all the mathematical transformations by which the application of the formula (for these stresses) may be made easier: ....and for convenience's sake we may continue to apply to the quantities occurring in this integral the name of stresses. Only, we must be aware that they are only imaginary ones, nothing else than auxiliary mathematical quantities." The dematerialization of the ether, when it was once found that light and heat still managed to come to us from the sun although its medium of transmission

had received such a rude shock, has progressed rapidly until to-day many accept the postulate that there is no difference between absolutely vacuous space and the ether except that the latter is the temporary seat of radiant energy and possesses a light vector. If this last definition of the ether means anything in a concrete sense, it implies that light has the power of changing a vacuum into an ether. When we stop to think that a vacuum means negation of everything, we realize what extraordinary things light and the ether are in modern physics. Just consider this statement of Professor Einstein, which is considered almost authoritative, "the places in space where these electro-magnetic actions (i. e., light) occur are here considered not as states of a sort of matter, but as self-existing things which are similar to ponderable matter and in common with it have the characteristic of inertia." One has merely to ask, what becomes of these self-existing things when light ceases to go through a certain space and it slips back into its state of vacuity?

Without going into details, we find a like trend in the hypothesis of the nature of matter. It began with the atom, as a minute simulacrum of ponderable bodies and then we proceeded to strip away one concrete attribute after another until for a brief interval matter was a manifestation of an entity, electricity. But even this idea was too concrete, too material, to serve and now the objective universe is the symbol, energy.

If we finally subscribe to these ideas, are we not really acknowledging that hypothetical science has failed as the interpreter of an objective world? It seems to me the guides to knowledge are now to be found in those subjective impressions which must depend on the individual and vary with him. Scientific laws are thus not facts to be discovered but the temporary consensus of opinion of a number of individuals who, for the time being, find them-

selves in agreement. The classical natural philosophy of Newton and Galileo has drifted into the transcendental symbolism which is apt to occur when German thinkers become the leaders in philosophy.

Evidently an almost chaotic condition had developed in our ideas of space, matter, and electricity. Each theorist advanced an hypothesis of the most tentative character which all felt to be inadequate. Yet in spite of the confusion, three points of agreement may be noted. In the first place, the ether as a material substance was impossible and even useless; the name, it is true, was left but it expressed now but the shadow of a reality. When the ether is called an electro-magnetic something, or a space differentiated only by the transient presence of energy, the name may signify something but it is hardly intelligible except as a mathematical symbol. Secondly, these theories agree in assuming electricity to be an entity; that is, we are to suppose that it is not due to a state of matter manifested by a special mechanical force, but a substance like matter, divisible into atomic elements and having inertia. Thirdly, the mass inertia of matter is not an invariable quantity but changes with the velocity of a body; thus it is the measure of matter only while there is no motion.

Such was the state of physical theory when Professor Einstein announced, in an article published in the *Annalen der Physik* for the year 1905, a new scientific principle which he believes will reconcile the contradictions of our new hypotheses; satisfy the three conceptions mentioned; and correct and amplify the Newtonian mechanics so that it will again harmonize with modern science. Whether this "principle of relativity" will accomplish all that its discoverer hopes can be decided only after an exhaustive trial. But there is no doubt as to the startling effect it has produced on scientific theory nor of the far-reaching importance of its conclusions. For example, Professor

Planck is said to have declared that this new conception of time and space surpasses in boldness anything that has appeared up to the present time in speculative science.

The name, "principle of relativity," is derived from Professor Einstein's first postulate, which is as follows: The idea of absolute rest or absolute motion is an impossibility to the human mind, and not only in mechanics, but also in electrodynamics corresponds to no properties of phenomena.

In addition, he announced as a second postulate that the velocity of light, V, in empty space is an absolute constant of nature. Hence this velocity, contrary to all others, is independent of the motion of the body emitting and the one receiving the light. The startling nature of this postulate is readily understood when we remember that the closest analogue to light is sound, the velocity of which is known by experience to be dependent on these quantities.

He has recorded for us, that he became convinced of the necessity for these postulates, because the theory of electrodynamics developed by Maxwell leads to an asymmetry, when applied to moving bodies, which is not true experimentally. For instance, when a magnet and an electric conductor are moved with respect to each other, the phenomena of the forces developed are observed to depend only on the relative motion of the magnet and the conductor, but Maxwell's theory requires a different explanation according to which is moved and which remains at rest. Besides this discrepancy between observation and theory, experiments, devised with sufficient accuracy, fail to show any effect of the earth's motion through space on the phenomena of light.

It will be convenient to grant these postulates and to follow Professor Einstein's deductions before attempting any criticism. In the first place, their adoption does away with the possibility of an ether and revives the conception

of space as a vacuum. Both theory and experiment show that an ether must be in absolute rest with respect to the motion of the earth, and the first postulate denies the possibility of absolute rest and motion. Again, the failure of Newtonian mechanics lies in the assumption, which is always tacitly made, that moving bodies are subject to the same mechanical laws as those at rest. As I have already pointed out, the modification to be made in the laws for ordinary bodies is very minute and would probably never have become of importance if modern theory had not been interested in the properties of bodies moving supposedly with velocities approximating that of light.

The root of the error in the mechanics of moving bodies, Professor Einstein believes, lies in our determination of time, and clear thinking in regard to time is nearly all that is necessary to clear up the trouble. Suppose a body, or a material point to be at rest relatively to a coordinate system of three rectangular lines, then its position can easily be determined by the ordinary geometrical method of measuring its distance from each of the lines by rigid measuring-rods. But if the point is in motion with respect to the reference system, its position depends on time and cannot be determined by the geometrical method. Our idea of time is usually defined by what we call isochronism; we say an event occurs at seven o'clock, when the occurrence of the event and the position of the hour hand of a clock at seven are simultaneous. But suppose the clock were at some distance from us, then we could observe the hand to be at seven only after it had passed beyond that figure, since it would take some time by any method of transmission for the intelligence to reach us. As the velocity of light is the greatest of all known motions, the least discrepancy would be caused by using light signals as the mode of transmitting such intelligence. Also if we accept the second postulate, our intelligence will be still more

accurate because the velocity of light is unaffected by other motions and we thus avoid the difficulty caused by the question whether any relative motion between us and the clock is an approach or a separation. To illustrate further our confusion as regards the measurement of time, suppose two persons wish to record two events, which we shall grant to be simultaneous, occurring at different places, A and B. An observer at A records the event when it occurs at A as being at T_A time, and instantly signals the fact to an observer at B. Whatever the means of signalling, a certain time will elapse before the observer at B is cognizant of it. For the reasons given, we shall adopt light signals as the best method. B observes the signal as T_B time by a clock placed at B and immediately reflects it back to A, who receives it at T'_A time by his clock. Although we have granted that both events were simultaneous, it is evident they will not be so recorded by the two observers. B will record the event at A as being later than the one at B. But if the velocity of light be absolutely independent of all conditions, then we should find that the differences of time going and returning are the same, or $T_B - T_A = T'_A - T_B$. This relation, if satisfied by any two events occurring at a distance from each other, is Professor Einstein's definition of simultaneous events or isochronism. The definition may be put in this form: Two events are simultaneous, if the difference of time to flash a signal there and back is equal to twice the distance between the two positions divided by the velocity of light, V.

Not only does this limitation in our inability to measure time affect our ideas of time, but it also has an important and unsuspected influence on our ideas of the size and shape of a body. This may be shown by an example. Suppose we wish to measure a rigid rod of length, l, moving in the direction of its axis with a velocity, v, then there

are two methods of measuring this length and they do not give concordant results.

First an observer may move with the rod and measure its length by applying to it a measuring-rod. He will evidently obtain the same result as if both he and the rod were at rest.

A second method is possible and is frequently employed. An observer remains at rest and notes the positions of the two ends of the rod, at a certain time, t, which he determines by means of clocks, also at rest and tested for synchronism. These two points are dependent on our ability to record simultaneous events; if the rod were at rest the problem would be the one already discussed, but in this case the rod has a velocity, v, and consequently the time used in signalling in one direction involves, $V - v$, and in the other $V + v$. So when he measures the distance between the two points, which may also be called the length of the rod, he will find it not equal to l as determined by the first method, if he still regards his clocks as isochronous; or if he determined two positions whose measured distance is l then his clocks will not be isochronous.

Now our measurements of length are usually made under the condition that we, clocks, and rods are all moving with the earth and so relatively at rest. Our measurements of length of bodies on the earth are thus different from those made on the same bodies by an observer stationed with his clocks in the moon. Also if an object on the earth is measured while it is moving with respect to the observer, it will have a different length from that which it has when relatively at rest. Thus the dimensions of a body are dependent on its velocity, and Newtonian mechanics, which assumes the contrary, must be modified when applied to bodies in motion.

Professor Einstein then derives a set of equations which will express the dimensions of a moving body as they

appear to a stationary observer. These show that the length of every body moving with a velocity, v, is diminished in the direction of its motion by the fraction

$$1/\sqrt{1-(v/V)^2},$$

and its dimensions at right angles to its motion are unchanged. Thus a sphere in motion becomes an ellipsoid flattened in the direction of its motion by an amount equal to the above fraction. It is proper to say, that this effect is quite inappreciable at ordinary velocities. This is true even of so great a velocity as that of the earth around the sun. This velocity is about thirty kilometers per second while V is 300,000 kilometers per second. One diameter of the earth would thus, to an observer in the sun, appear shortened about 7.5 centimeters, or three inches. But this effect becomes quite important for velocities approaching one-tenth of V. And a velocity equal to that of light is absolutely unattainable because the length of the moving body would then be reduced to zero. So we have the curious anomaly of a finite velocity producing an infinite effect.

The same equations also show that if one of two clocks, which are synchronous when at rest, be moved with a velocity, v, the stationary clock will run faster each second than the moving one. The maximum value of the difference is when one clock runs infinitely faster than the other.

We must not lose sight of the fact that this discussion, so far as mechanical bodies and motions are concerned, is purely academic, because for them the ratio v/V is too small in any known case to have an appreciable effect. The principle of relativity has its greatest significance when applied to problems of electricity, provided we accept the theory of electrons. If the atom of matter be composed of particles of electricity, if the inertia of matter be variable and due to electrical reactions, and if the velocity

of the electron approaches that of light, then the effect of its velocity on its shape, size and mass is an important matter. For, let m be the mass of an electron at rest, from Professor Einstein's formulae, its apparent mass in the direction of motion is

$$\frac{m}{\sqrt[3]{1-\left(\frac{v}{V}\right)^2}},$$

and its mass at right angles to the motion which would make it resist change of direction is

$$\frac{m}{1-\left(\frac{v}{V}\right)^2}.$$

Thus we have the unusual result of not only a variable mass, but also a difference in mass according to the direction considered. There is a close analogy between this apparent increase in the inertia of a moving electrified body and the increase in the inertia of a body moving in a fluid. In the hydrodynamic problem, we do not find it necessary to consider the real mass of the body as variable, since we can attribute its apparent variation to the energy required to set the fluid in motion. We may, in a similar manner, for the electrodynamic problem, ascribe the apparent increase in mass to the energy necessary to overcome the reactions of the electromagnetic field through which the body moves, and thus keep the mechanical mass constant.

Another consequence of the hypothesis is that mass also depends on energy. This result is of extraordinary importance, for it means we have no ability to distinguish between the inertia of a physical system of bodies and its energy; in other words, between the inertia of a body and the energy content of the space surrounding it. When we recall Professor Einstein's conception of space and energy, which I have quoted, most persons will become

convinced that no idea of the nature of matter could be more abstract. The mass of a single body thus remains constant only when its energy remains constant. If it gives out heat, light, or electromagnetic energy, or if it even moves, its mass continually decreases, until, theoretically at least, it would melt away into a complex of energy. I know of no name to designate so immaterial a thing as this complex of energy, located somewhere in vacuous space. To be sure the interchange between mass and energy is very slow, so slow that ordinary minds grow skeptical of its existence. Thus a body radiating enough heat to warm a kilogram of water, one degree Centigrade, would decrease in mass only about 4.6×10^{-11} grams. It is fortunate that mass dissipates so slowly or we should indeed be things dreams are made of.

The last deduction I shall note is that the temperature of a moving body is less when measured by a moving observer than by one who is stationary.

Such are some of the more important conclusions which have been derived from the principle of relativity. If they can be established and men can be persuaded that the universe is, or even may be, as they indicate, a revolution has been accomplished in scientific thought. The postulates are apparently so simple and at the same time so bold, the conclusions are deduced by such rigorous mathematical development, that we must give the highest praise to this memoir in which so many of the perplexities of modern science are attacked. If we are to criticise the conclusions of this hypothesis, and they are opposed to what seems reasonable, it must be done by examining the postulates; in them, as in all hypotheses, lies the real strength or weakness of the system.

Professor Einstein asks us to accept two postulates. The first is: We have no ability in us to determine abso-

lute rest or motion. We can say only that one body is at rest or changes position with respect to others. Phenomena are conditioned entirely by the relative positions of bodies and we can gain nothing toward this explanation by introducing the idea of absolute position. The second postulate is: The velocity of light, V, is a universal and absolute constant.

Apparently no objection has been made to his statement that these two postulates are sufficient from which to derive all the conclusions mentioned. But it seems to me that at least two more independent postulates are advanced in the memoir on relativity, which must also be granted. I shall propose as a third, Professor Einstein's definition of time; and as a fourth, the assumption of the atomic nature of electricity. The fourth postulate carries with it as corollaries, that the amount of electricity per atom, or the electron as it is called, is a universal constant, and that the mass of the electron is variable. My excuse for offering them is, they cannot be derived from his postulates, and his conclusions require them.

The first postulate can be granted at once and unreservedly for all purely mechanical motions of bodies. These involve merely changes of position and do not affect the nature of bodies nor their phenomena. But it must be accepted with limitations when in addition to mechanical motion (and by mechanical motion I mean a mere change of position) one body is emitting energy in the form of heat, light or electricity, and the other receiving it. True relativity requires a strictly symmetrical arrangement. Now the mechanical motions of two bodies with respect to one another are symmetrical. If a body has a velocity, v_1, to the right and another a velocity, v_2, to the left, they approach each other with a velocity, $v_1 + v_2$. The same result is obtained if the velocities are interchanged. This symmetry is not the case if the first body is emitting en-

ergy and the second receiving it. The behavior of each is then conditioned by the behavior of the other and in an asymmetrical manner. As an illustration: Let a body, by a periodic disturbance of its parts send out a wave of heat; this proceeds, according to theory, through space in all directions and on encountering a second body produces a periodic disturbance in it. The effect of this energy is twofold; the motions of the parts of the two bodies are symmetrical but the wave itself does not proceed from the second body in all directions but only in the one already impressed upon it by the first body. To make this clearer, we may start a wave along a stretched string by plucking aside one part of it; the wave will travel along the string in two direction, causing successive parts to vibrate symmetrically, but these parts will send the wave in one direction only. So we may say the velocities of two bodies are relative because the quantities appear in the mathematical expression in a symmetrical manner; so also are their momenta or their masses multiplied by their velocities. But this is not the case for energy since it involves the square of the velocity. The squared velocity remains positive although we change direction and the composition of two energies is always an addition. For true relativity, a change in direction must be accompanied by a change in sign. The extension of the idea of relativity to involve cases of radiant energy is partly responsible for the conclusion that mass is a function of energy.

Professor Einstein's second postulate has been widely discussed and many attempts have been made to interpret it. At first sight it seems absurd to say that the motion of a body emitting light does not affect the velocity of the light emitted. But the fact remains that all our experiments, and they have been most accurate and searching, fail to find any difference in the velocity of light whether the sources and the recipient are at rest or in

motion. As we have seen, Professor Einstein, with admirable directness, goes straight to the point. He accepts the reliability of these experiments; assumes the constancy of V as a postulate, determines its effect on the space-dimensions of bodies and on time, and finds that we must change our previous ideas of these radically. Let us, when discussing V, consider it under two heads: first, when light passes through transparent material bodies and is subject to experimental verification; secondly, when it passes through space absolutely deprived of material bodies and so is not subject to experimental evidence.

In the first case, we know that there is a true path and that light moves very approximately in straight lines, and we have quite accurately measured the time light requires to move from one place to another through various substances. V is here the length of path divided by the time —a true velocity. We know that this V is a variable; it is less in water than it is in air, and still less in glass. It is also less in dense air than in rarefied air. Not only does V vary with the kind of matter through which light passes, but it also depends on the motion of the medium as shown in the experiments on the velocity of light passing through columns of moving water made by Fizeau and repeated by Professor Michelson. The velocity of light in material media is therefore subject to the variations which influence the velocity of sound and other types of motion and evidently cannot be the V assumed by Professor Einstein to be an absolute constant.

He must then limit his postulate strictly to what is called the velocity of light in absolutely immaterial space. There are two methods of obtaining this value. We observe the difference in time between the calculated eclipse of some satellite and the recorded observation of the event or we use other stellar phenomena. If we know the distance and the time, we say the velocity of light in empty

space is the quotient of the two. Such observations are quite inadequate to settle experimentally the question of the constancy of V. Our measurements of the distances are crude for such a purpose, and also the light must travel part of the way through a material such as the air. Neglecting such astronomical methods, we have left only determinations of V made on the earth where the path is necessarily through matter. Here, what we are to call the constant V can be found only by extrapolation. For example, we find that the ratio of the velocity in air and in water is about four to three and agrees with their refractive indices. We find also that the velocity in different gases as they are reduced in density, tends to a common value, which is independent of the kind of gas. If we take this extrapolated value of 3×10^{10} centimeters for the velocity of light in empty space and assume the refractive index of space to be one, then we may calculate back and by this method we find the absolute refractive index of air at ordinary pressure to be 1.0002. Such an experimental method and such reasoning are highly unsatisfactory. In the first place, we start with measurements of lengths and time which are subject to experimental verification and calculate V; from these we pass to the supposititious V where only one factor, the time, is subject to experimental proof and the other factor, the length, is not, as we cannot experiment on the path of light in free space. But passing this objection, we have calculated from length and time determinations a value for V and Professor Einstein declares it to be an absolute constant. He then reasons backward that the length of a body and time determinations must be variables with motion, in order to keep V constant. Of course, if Professor Einstein wishes to look on the universe as a purely abstract conception, and if he wishes to make what we sometimes call concrete or objective phenomena correspond to his preconceived idea,

he *can* assume V in a vacuum to be a universal and absolute constant and no one can disprove it scientifically. But there are some who still cling to the idea that dimensions of bodies and time are not subject to our fancy, and who believe that if any quantities must vary, it should be those which we have no means of determining directly. And after all it is asking a great deal of us, to upset our ideas in order to explain at bottom a single experiment, that of Michelson and Morley, however accurately it has been performed and however puzzling its results may be. He has not even the justification his predecessors would have had. When the ether was believed to be a crystalline solid which vibrated with the passage of light rays, V had a real meaning although we could not determine it directly. When the ether was a substance which periodically varied electromagnetically, we could still say that V had a possible meaning. But the latest definition, that the ether is absolutely quiescent space to be distinguished from vacuous space only by the fact that it is the seat of an entity, called electromagnetic energy, and contains a light vector, makes the word velocity when applied to V, absolutely without meaning in any ordinary sense of the term. To speak of a motion in an absolutely quiescent space is a rather startling statement. The ether also becomes a local affair shifting back and forth in vacuous space according as light is present or not. The feeble light of a candle, apparently, changes nothing into something.

But aside from these considerations which lie in the debatable land and which will be decided largely by the temperament of the individual, there seems to be an absolute contradiction between the first two postulates as I understand them. If V be the value of the velocity of light in an absolutely quiescent ether and itself a constant, then the velocity of light in a material medium, such as air, is an absolute velocity or motion when referred to V. Now

the first postulate declares that we can have no knowledge of absolute motion and, in addition, that the explanation of phenomena are not conditioned by absolute motion.

The third postulate, which I have proposed as necessary for the system proposed by Professor Einstein, is his definition of time. He says that our time is defined by synchronism or the simultaneous occurrence of an event and the position of the hour hand of a clock at a certain position, the number seven for example. This is certainly not our idea of time but merely our method of measuring it quantitatively. If we did not have an adequate idea of time as the mere succession of events we should have no conception of what simultaneous occurrences are. His definition gives me the feeling that if I could make clocks go slower my life would become longer. In the review of his memoir, it was pointed out that his definition of simultaneity or isochronism, when combined with the constancy of V, led to the conclusion that two clocks which were synchronous when relatively at rest, would not remain so if one of them were given a velocity. Moreover the length of a body in motion decreases with respect to its length when at rest. It is certain that these results depend on the constancy of V. If we are willing to rest our knowledge of time and space on a value which by the nature of things can never be put to a direct test, I am convinced of the theoretical correctness of these results. But I am not willing to do so. I prefer to trust to the invariability of time and space phenomena, even if it requires V to be a variable, and time measurements to contain an unavoidable discrepancy. By so doing, I am aware that I may forfeit an explanation of the Michelson-Morley effect and of some others, but this loss is not overwhelming and we can afford to wait for an explanation until a future time.

I am the more willing to take this attitude, because I can see no means of applying the principle of relativity to

the test of experience. As we have seen, the velocity of light in any material medium is a variable and can be no more trusted than the velocity of sound.

Perhaps this illustration may make my meaning clearer. Suppose a race of men to exist who are blind and have no knowledge of electromagnetic radiation, but who wish to measure the lengths of moving bodies. They will undoubtedly be compelled to get this information and that of the synchronism of clocks by sound signals. It is evident, that observations carried out, under conditions similar to those imposed by Professor Einstein, would indicate that the length of a moving body underwent changes. And while they could make corrections for some of the effects, because sound waves are largely affected by the motion of media and of sonorous bodies, yet they would undoubtedly come to the conclusion that the dimensions of a moving body depended to some extent on its motion. Now, if we bestow sight on one of these men, he would be able to correct their measurements, because by his immensely more rapid light signals he could gain a much more nearly instantaneous value for synchronism. We are, at present, in the condition of this man. As we improve in our ability to measure the velocity of light under different conditions we shall, Professor Einstein thinks, get closer to the knowledge of the absolute V and to the relations for space and time which he has derived. But we may suppose men will some day find a kind of radiation which has a velocity greater than V and by its aid remove the conviction remaining in our minds that motion affects length and time. Calculation may show that material bodies cannot attain this velocity, but we are speaking of an immaterial radiation. To say that such a radiation is impossible is as futile, at least as unscientific, as for a race of the blind to say that there is no light; especially so,

as it is possible that gravitational attraction may involve a velocity of transmission vastly greater than that of light.

Since the motion of any ponderable body is too slow to make the ratio v/V an appreciable quantity, the only supposable case where this ratio can enter as a determining factor is in problems of radioactivity and the discharge of electricity through gases. In these, the particles of matter are supposed to be so small and to have a velocity so great that their mass and size are functions of their velocity. But to limit the applicability of the principle of relativity to such supposititious bodies as electrons, is to rob it of much of its importance, and we should hardly consider it one of the great principles of nature.

But even this is not all. If we wish to apply the principle to electrons, we shall need a fourth postulate. We must assume that a quantity of electricity is atomic in character; either the charge of electricity associated with what we call an atom of matter is an invariable, or, as it is now usually expressed, electricity is an entity which may be divided into invariable, equal, and indivisible parts called electrons. As a corollary to this postulate, the ponderable mass of an atom is a variable depending on its velocity. It will suffice to consider this postulate very briefly. It may be shown at once that the constancy of electric quantity and the variability of mass is not a necessary assumption. All experiments, which involve both these quantities, include them in the form of a simple ratio, e/m; where e is the quantity of electricity and m the mass of the electron. It is evident that any value may be given to this ratio by supposing either one of its members to remain constant and the other to vary, or by supposing both to vary in opposite fashions, and it may be shown that it is just as reasonable to keep m constant and to let e vary with the velocity as to make the contrary assumption. This is certainly possible until we have experimental evidence which

will determine our decision and this evidence is not likely ever to be at our disposal.

If we attempt to estimate the results which follow from the postulates of relativity, the first would be that Professor Einstein has proved that we cannot theoretically measure in space and time moving bodies exactly by the laws which apply to bodies at rest. The first postulate will be accepted for all mechanical motions and no further attempts will be made to find experimentally an absolute motion. But the second postulate contradicts the principle of relativity, if by V we mean the absolute motion of *something*—matter, energy, or light through quiescent space. If, on the other hand, V is understood to be the velocity of light in space containing matter, then it cannot be taken as a universal constant. The third postulate concerning time should not cause us to change our belief that the dimensions of a body and the unit of time are independent of velocity, but it should show us how to correct our measurements of moving bodies, as we must correct all subjective measurements of objective phenomena.

If we grant the assumptions of Professor Einstein, the theory of relativity is a perfectly logical system so long as we apply it to abstract systems moving with a constant velocity in a straight line. But I can find no evidence that the postulates agree with experimental facts and certainly, to the present time at least, its conclusions will not affect experimentally the laws of mechanics. Also its theoretical conclusions are limited, as yet, to the rare cases when there is neither any change of speed nor of direction. We should remember before we abandon or modify Newtonian mechanics that mathematicians have devised numerous systems which point to different mechanical laws. If we postulate a fourth dimension to space we can deduce perfectly logical laws for a mechanical system that is quite different from the laws of ordinary mechanics. And the universe,

constructed on this principle, is a very beautiful and interesting one. It is the privilege of mathematicians to deal with symbols; the physical universe is no more important to them than any other universe which can be developed symbolically. The trouble occurs when the distinction between the real or physical universe and the symbolical or metaphysical universe is obliterated in the minds of men of science.

Louis Trenchard More.

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PURPOSIVENESS IN NATURE AND LIFE.

"Et rien, afin que tout dure,
Ne dure éternellement."
—Malherbe, *La Prise de Marseille.*

AN ordering process amid the ceaseless flow of things has been observed since the beginning of thought about them, and is still the chief material of our anthropomorphisms, gross and refined. We may neglect the flow in our effort to attend to the things, or ignore the things in our overwhelming sense of the flow, but sooner or later the conviction is borne in upon us that nature has some mode of moving or acting, some way of dividing or uniting, which can take none of its wonderful potency from the mere peradventure of chance. How are we to explain the adjustments and adaptations which, in both inorganic and organic, present the world to us as everywhere shot through and through with mind? Why should the universe, like the intellect, segregate and classify, bringing likes together and separating or assimilating unlikes? What is it that matches the rounded boulder with the sphered planet, and shapes the orbits of worlds and the orbits of electrons on the same plan; that relates the spiral of the nebula and of the shell to the volute of the Ionic capital, and these to the curl of the wave as it breaks on the shore; that passes from regularly spaced ether pulses to regularly spaced air waves, sand waves and ocean rollers, to re-appear in the iterated lines of decorative art; that makes one the geometry of the crystal and the geometry

of the honeycomb, the intelligence that moulds the dandelion seed to the wind and the intelligence which builds up human industry, navigates the seas and explores the heavens?

Underlying all anthropomorphism, whether it frankly assimilates nature to man's bodily characteristics, or more subtly reads into it his will and mind, has been the sense that we are in some sort one with the universe and are entitled to fashion it in our own image. But we have selected for our material of interpretation not the deepest and hidden characters of our being, but rather the things most obvious to us as sentient organisms. Our very idea of intelligence is so bound up with conscious and voluntary experiences that these have become for our thinking the *sine qua non,* if not the very essence, of so-called purposiveness. Are our own ends consciously realized? Is human planning inconceivable save as the outcome of consciousness and will? Then all arrangements that suggest contrivance by being directed to end must be similarly determined. Regarding intelligence as constituted of its conspicuous instead of its fundamental characters, we thus commit ourselves to the task of finding will and consciousness in the world at large. Here is the royal and easy road to the elucidation of "design in nature." In this view there can be no purposiveness save as the result of the sentience we associate with mind. But in the deeper insight we are emancipated from so embarrassing a superficiality. Instead of being rounded off by consciousness and will, intelligence is there seen to be the ways in which we act and think, not our awareness of those ways; so purposiveness, whether highly complex, as in organisms, or relatively simple as in the inorganic, is recognized as not primarily feeling, consciousness, or will as such, however closely it may come to be implicated with these, but primarily a mode of motion, a method of change, which has its

roots in the very nature of power. Out of the process thus deeply grounded come the forms that are purposive because they endure, and endure because they are purposive. Change may indeed be the unmaking as well as the making of form, yet nature's richly stored workshop shows us everywhere a final triumph of the ordered over the fortuitous tendencies. The fundamental character of change turns out to be not merely a continuation of the "immobile," but the setting up of form; the fundamental character of form manifests itself not merely as "a snap-shot view of a transition," but as an end or result which is reached and maintained. It is vain to tell us that reality is only the flux of things, and may be summed up as the ceaseless upspringing of something new. What we confront is a vast cosmic drift towards purposive configurations, towards intelligent adaptations, towards symmetry, beauty, harmony, and therefore towards good.

DYNAMIC INTELLIGENCE.

Now the trend towards forms which are purposive because they endure, and endure because they are purposive, is writ large in the visible phenomena around us because it is first writ small, yet all the more potently, in the ground of things. The immediate source of this drift is the so-called conservation of energy, applied as a doctrine in physics to moving bodies; yet maintenance thus conceived is only the more obvious and superficial aspect of a conservation wide and deep as the cosmos itself. Beneath all the vicissitudes which submit themselves to the ordering process is the endurance of that which changes; earlier than motion is the conservation of the things which move and of the ether or power system from which they emerge and with which they are one. If the ether be granular, as there is reason to believe, the universe accessible to knowledge must be envisaged as a unity of dynamic oppo-

sites, of ultimate elements stressing and stressed, each requiring all and all insisting on each. Summated as part contributing to whole, as whole arrayed against part, this insistence on the one by the many emerges as an ever-present and everywhere exercised function of cosmic self-conservation. Before the appearance of matter, maintenance is effected without translatory motion. As likes, holding and held, the power elements are in equilibrium; the antagonism which they exert and to which they are subject is the same for all of them throughout the cosmos. Stresses and re-stresses are everywhere balanced. There is everywhere incoming and outgoing strain kept from change by likeness, and only awaiting difference to manifest itself as motion. And translatory motion begins when, by some modification of the units—possibly by partial coalescence of them into definite groupings—ether gives rise to matter. Were only one material unit formed it would not move, since the stress on it would be the same from every direction. But when many such units are produced, the relation between them and the system which we call gravitative springs up. It is not inconceivable that they shield each other in some way from a total stress which acts on all; but whatever the explanation, we must assume that the pressure inward from without all such bodies is greater than is exerted by the ether system outward from within them. And with the advent of matter and of matter in motion comes the drift towards forms and configurations that endure.

All kinds of motion in the universe may be traced to differential stresses whose goal is equalization, and therefore endurance. Inorganic matter does not move "of its own accord." If the stresses on a body from all directions are equal, it will remain at rest; if it be stressed more from one direction than from all other directions it will move. Set in motion where there is no resistance, it continues to

move, and the energy expended in setting it in motion is conserved for future delivery whenever resistance is encountered. Where motion occurs through a resisting medium or against resisting objects, the initial energy is distributed as motion through the resistances. As the energy of moving bodies is a secondary form of the constitutive cosmic energy, so motion is a secondary form of cosmic self-maintenance, since it secures the continuance, either as kinetic or potential, of the energy expended in producing it. Power is two-sided: the unit of it must everywhere have against it an opposing, but at the same time a supporting universe. So in the realm of motion, there must arise instantaneously or ultimately re-stresses equal in amount to the originating stresses. The universe is a self-complicating and a self-ordering because self-maintaining system: it must develop the differences which are its superficial and temporary character; it must insist on the likenesses, on the two-sidedness, which is its fundamental and enduring character. As motion is a way or means by which differential stresses sooner or later automatically eliminate themselves, all change may be regarded as a procession towards conditions of equalized stress. In this way single bodies under differential stress reach positions of rest by motion through their resisting environment; in this way moving configurations, from atom to solar system, have their differentials automatically equalized and harmonized. Where stresses are not equal, change proceeds until by motion away from the direction of greatest stress towards the direction of least stress the collocations, configurations and forms which endure are set up. Objectively regarded, these are ways or means to endurance, and it is because they realize the end of endurance that they display "intelligent adaptation" character, suggest purposiveness, give rise to the idea of contrivance or plan, seem put together that they may continue. The link be-

tween contrivance by man and purposiveness in nature is thus the link of an underlying process common to both and out of which both arise. Due neither to consciousness nor will, intimately as it may come to be bound up with these, intelligence is fundamentally dynamic, belongs to the inorganic before the advent of life-forms, and as modes of motion directed to end, culminating in maintenance, is rooted in the very nature of power.

TELEOLOGY IN THE ORGANISM.

The same drift towards endurance reappears in the organism, not as totally new, but as fundamentally old. The principle of continuity would alone entitle us to regard the living system as derivatively modelled after the cosmic system from which it emerges. What we find in the transition from inorganic to organic is passage from the non-living material unit which is universe-maintained to the vital aggregate which maintains itself, with the function of conservation complicated, but not obscured, by the peculiar conditions under which matter ascends to protoplasmic rank. The electrons, atoms, molecules of an inorganic body are held in existence by the universe—do not depend for their "properties" on the material aggregate which they unite to form. A molecule of iron remains a molecule of iron however the size of the mass to which it belongs may vary. But in living matter these elementary parts, built up into complex units highly sensitive to the inner environment which all of them form for each, have their functioning and structure determined by the nature and activities of the system as a whole. Made up of interdependent elements, each necessary to all, all insisting on each, the one everywhere contributing, the many everywhere dominating, the organism is committed by its very nature to self-maintenance. The differential stress which ensures motion towards the end of endurance is thus

exerted as the power of the whole; the path of least stress for both unit and system is the direction leading towards activities and structures that secure self-maintenance. That the organism does not merely "hold together," but comes internally and relationally to be motile, results from expenditures of energy for self-maintenance and from the involved need of replenishment from without. Repair of structures metamorphosed or injured flows directly from the impulsion to self-maintenance. Any inroad upon normal completeness brings a tension to bear on the whole system, and the organism finds it easier to relieve that tension by repair than to reaccommodate its whole structure to the changed conditions. It is only a more intense, because organized, form of that strain which is seen in reproduction by means of specialized germ-cells developed to secure endurance threatened by the inevitableness of death.

How is the organism guided to the intelligent adaptations which insure self-maintenance? What gives it the semblance of a series of appliances, machine-like, intelligently contrived, for doing the work of life? How does it come to suggest a subtle arrangement of parts put together for a purpose? Man fashions a tool or machine because he is aware beforehand what shapes or collocations will lead to the desired results. The organism, which works unconsciously, has no such prevision based on experience. Its process must be one of effects achieved from instant to instant. How can it reach end by means adapted to secure end? We find the answer to this question by realizing to ourselves the meaning of that original set of the organic system towards self-maintenance which has been either ignored or naively taken for granted in the theory of natural selection. The "contriver" is neither an inner consciousness obscurely or clearly aware of the right means, nor an outer intelligence working as artificer in

relation to his machine, but the system itself plus the motion process. The "plan" in accordance with which the intelligent adaptations are to unfold is implicit in the organism at the very outset of its career. The system has had organic property imposed upon it—the property of life—and is under impulsion to work only in ways that secure self-maintenance. And the impulsion is not due to a "vital force" mysteriously permeating the organism and urging it to intelligent action. It proceeds from the organic units summated as a whole, the one subordinated, the many dominating, and all making an inner environment for each. It is an internal stress of the system on its parts, a dynamic thrust through which each unit and group of units finds what it must do marked out for it by physical influence passed from unit to unit and exerted upon every unit by its immediate environment of units. In this sense, but in this alone, the determination to self-maintenance and to self-maintaining activities is projected forward, as it were, by the nature of the system, just as, though in a far less complex way, the "affinities" of the atom are projected forward by the nature of the atom. In this sense the easiest path for organisms and organic parts is predetermined. Impelled to self-maintenance, it imposes subfunctions of maintenance on the parts, and these in working, however inadequately at the beginning, are guided by the process which automatically compels change as long as there is differential stress, and automatically ceases to compel it when conditions of equalized stress have been attained. It is out of this impulsion of the whole to self-maintenance, this imposition of vital, end-reaching activities on the parts, and this subjection of system and parts to the motion process, that tissues and organs gradually arise. In their beginnings they work against friction, with much waste of power and failure of efficiency. But the resistances they encounter operate as dif-

ferential stresses, and it is by the impact of these that the adaptations are gradually perfected and acquire "intelligent" because efficient form. As long as there is an attainable economy which is not reached, change goes on by motion away from the direction of greatest stress towards the direction of least stress until the appliance or organ has acquired its efficiency maximum. Intelligent adaptations in the organic realm, given the nature of the organic system as a self-maintainer, are thus products of differential stresses that have automatically eliminated themselves by motion. They are enduring forms just because of this elimination. They are "intelligent" because they are ways by which self-maintenance is secured, and not because of their need of any conscious or psychical element to make them intelligent. Certain ends of the organism come finally to be secured by conscious, voluntary action, but even for these there is the same underlying nature of the system impelling to self-maintenance, and the same resistances to action functioning as moulders of intelligent adaptations. As the time-and-space resistances which oppose themselves to our use of a blunt tool lead to the sharpening of it, so do such resistances work to the perfection of our machines and the facilitation of our enterprises. In organism and universe, then—in the one because in the other—self-maintenance is the function of the system as a whole, and differential stresses, working through motion to set up equalized stresses, determine how self-maintenance shall be achieved. For the narrower view of it the process seems mechanical; in the wider view it is seen to be teleological. Causality and teleology, in other words, belong together. However subject to delays and reversals, the teleology of the cosmos is an elementary anticipation of the more complex and precise teleology of the organism. However disguised by consciousness and will, the teleol-

ogy of the organism does but expand and write large the cosmic drift towards enduring forms.

DYNAMIC MEMORY.

Thus far we have dealt with organic development in its phylogenetic aspect. Let us now consider the ontogeny of the system. Why does endurance necessitate reproduction, and why does self-maintenance show itself in the form of heredity? How does it come about that, in the phenomena of growth, the parent or parents should be imitated, recapitulated, and with such a degree of exactness as to suggest conscious and voluntary mimicry of the past? What if it should turn out that this form of memory is neither conscious nor voluntary, but purely dynamic? For answer to these questions we need to examine the reproductive process at its very origin. For life will be found to begin in the active resistance which the organic system offers, not to diminution, but to increase of its bulk. The first synthesis of protoplasm from inorganic matter brings not only increased complexity of material, but also organization of the elements aggregated. Under the domination of the system, configurations arise in adaptation to it and to one another. As the bulk of the whole goes on augmenting by further aggregation, the adaptation of the unit lags behind the structural character of the system, and this lag, in spite of slow modification, is a continually increasing value. The resulting tension between system and unit, the latter clinging to a structure required in the past, the former requiring a unit structure suited to the present, finally reaches a critical stage, whereupon the system divides into equal parts. By this division, which has many analogues in animal societies, the strain is relieved. But the products of the division do not remain quiescent. The hand of the past is upon them. Another tension, individual and collective, is now theirs—the tension of incompleteness.

Accustomed to a larger system, and made up of units more or less definitely adapted to it, they are now impelled to complete themselves, and to do this in a particular way. The time-and-space order of change with which they have been impressed summons them to the repetition of that order by definite and precise vicissitudes. Under stress from the dominating system, not as existing merely in the present, but as a time-and-space whole, they find it easier to build up the old conditions then to readapt themselves to the new. Continually receiving fresh material from the outer world, they not only mould that material into the accustomed forms, but also impress upon the elements assimilated the power to assist in further assimilation until the original system has been completely reproduced. The objective type or plan of the mature organism to be thus exists in the product of division or reproductive germ as a series of paths of least stress and greatest stress towards which and away from which it must move. Growth is the recapitulation of those experiences of activity and structure which, by interadaptation of the units, the dominating whole has made easiest for all of them. It represents the passing over of differential stresses into equalized stresses, involving changes of form along with changes of place; it is an aspect of end-reaching, the completion of the organism being an end for all the units and for each of them. It is therefore, objectively speaking, a means to self-maintenance.

The phenomena of heredity thus require us to regard the organism not merely as a structure in the present, but also as a process in time. The whole which at any particular instant determines to self-maintenance is at once structure-whole and process-whole. As the unit structures are interadapted and thus come to be insisted on by the system, so are their unit time phases interdependent and interdetermining: they constitute an order of change on

which the dominating system insists. And the method by which the time phases are enforced along with the spatial structures resembles memory just because the underlying process of memory is dynamic, and not in any sense conscious or voluntary. We are enabled to learn by heart because the brain units engaged acquire an order of change to which the whole brain system adapts itself, and on which, because of that adaptation—arraying the many against the one—the brain system insists. The "affinities" of the inorganic atom remain the same, whatever may have been its experiences; the behavior of the organic unit is determined by the experiences of the system to which it belongs. And the recapitulation of them in heredity results, not from the distribution of specialized elements to each area of the organism to be built up, but from the co-operating dynamics of the entire system as a time-and-space whole. The "gemmules" of Darwin, the "micellae" of Nägeli, the "dominants" of Reinke, the "determinants" of Weismann, and the "pangenes" of De Vries, are no more than symbols which look forward to the real explanation. Beneath and beyond them is the determination of the system as part and whole to unfolding and expanding in a particular way. Life is thus cyclical, and death manifestly results from the complexity and interdependence which underlie heredity, and from the heredity that makes insistence on the time-and-space order of change inevitable. The organism is not an object which would go on living for ever were it not attacked, harrassed, and finally worn out by the unceasing onslaught of hostile influences from without. It is a system carried through a cycle of change which, however that cycle may be lengthened, no improvement of the conditions of life will ever enable it to escape. Birth and growth, maturity, old age, death—these are the stages of a time-and-space order on which the dominating system insists, and to which the contributing

units are for ever subordinated. Reproduction is therefore a provision against the inevitableness of death—a means to self-maintenance required by the insistence of the system as time-and-space whole on the organic cycle. The elementary form of death is seen in simple protoplasmic fission; to this is added in the higher organisms the dissolution of the non-reproductive body or soma.

ORGANIC CAUSATION.

The need of envisaging the organism as a dynamic system, for the explanation of which neither vitalism nor mechanism avails, should now be obvious. In dealing with vital phenomena we are really dealing with what may be called organic causation — the determining and guiding power which the living system exerts over its interrelated and cooperating parts. And by the use of this term it is not meant to set up any absolute distinction between kinds of causation. All it involves is the recognition, not of different species, but of varieties of the same species—not of two wholly unlike sorts of the cause-effect relation, but of that relation as it shows itself, on the one hand in the inorganic, on the other when operative under organic conditions. Inorganic causation may almost be said to define itself by virtue of the fact that it is non-organic. The objects to which it gives rise in the pre-vital world exist as objects independently of one another. In the living body, and none the less really though less obviously in the social system, structures and activities reciprocally imply, require, are needed by, and insist upon each other, with the power of the contributing part subordinated to the power of the dominating whole. Facing nature and society, the human individual finds himself in the grip of two forms of control. On the one hand he is dealing with things which have no organic need of each other; on the other

he is dealt with by beings correlated, interdependent, forming a power system from which he receives and to which he must contribute. In inorganic causation we get a multitude of separate incidences no one of which is dependent on the rest. Organic causation is the product of numbers reciprocally involved, contributing yet subordinated, interdependent yet dominated—a power cumulative from unit to system and refluent from whole to part as a power of self-maintenance.

This domination of the one by the many finds elementary exemplification even before the advent of life. The inorganic unit is never indifferent to numbers; everywhere the one undergoes modification, if not change of property, from its association with the many. As part of an aggregate it is armed with the whole inertia of that aggregate; it then offers greater resistance to impact and moves farther than when acted on alone. And the modification begins earlier than with Ruskin's "little flake of mica sand" which rose from the weakness of isolation to be "knitted into a strength as of imperishable iron" as the citizen of a granite kingdom. Lothar Meyer[1] called attention to "the existence of a connection of some kind between the functions of the parts of a molecule and the composition—using this term in the widest sense—of the whole molecule." The action of an aggregate on its parts is well shown when, in the phenomena known as "elastic reaction," a wire which has been recently and frequently twisted develops a "sense of fatigue" that makes its behavior when again twisted different from that shown when it has been a considerable time at rest. If a bar of nickel steel, when put under a drawing strain, hardens that part of the bar which is nearest to the breaking strain, we are compelled to believe that it is the whole aggregate which, if only in an elementary way, resists the threatened rup-

[1] *Die modernen Theorien der Chemie*, pp. 6, 103.

ture. So in the formation of crystal, as Herbert Spencer[2] suggests, "we are entitled to conclude that the crystallization goes on in each part under the control of all the other parts"—that "the entire aggregate of crystals coerces the molecules in each place, while these in turn join all the rest in coercing those in every other place." Then, with a happy thought whose deep-seated implications he does not seem to have perceived, Spencer infers a similar coercive action exerted by the organism as a whole on its parts, remarking as he is about to extend the analogy, "The thing is done, but it is impossible to imagine how it is done." A pertinent suggestion of how it is done for the organism is given in *The Cell in Development and Inheritance,* where (p. 59) Edmund B. Wilson writes: "As far as growth and development are concerned, it has now been clearly demonstrated that only in a limited sense can the cells be regarded as cooperating units. They are rather local centers of a formative power pervading the growing mass as a whole, and the physiological autonomy of the individual cell falls into the background."

The foundation is the same on whatever aspect of the life process we concentrate our attention. Under inorganic causation the molecules have a merely inorganic relation to one another. They act as molecules of carbon, oxygen, nitrogen, phosphorus, sulphur and what not, and the configurations they will assume can be described in purely physical and chemical terms. But when favored by organic conditions they show the results of a new relation. What each of the molecules shall do is henceforth determined by what all of them must do. The single properties of the units, in a word, are transmuted or merged into a collective property, that of life itself. The so-called vital force imagined by an earlier age, the potency missed by the physico-chemical theories of later date, are just this

[2] *Facts and Comments.*

influence exerted by the dominating system over the contributing yet subordinated units, just this power of the all imposing collective character on each, just this superposition of organic property upon inorganic property, just this lifting of matter from simpler forms in which it is universe-maintained into complex forms that are self-maintained. The driving and directing power of life, buttressed in and derived from the pre-vital cosmos, is thus the organism as a whole. Life adds nothing to the store of energy present in any organic system, nor does death abstract in the least from that store. What happens in the one case, while organic conditions prevail, is the imposition of the law of the whole with which the units have to reckon; what happens in the other, when organic conditions finally cease, is the handing over of the system to the more elementary determinations. The difference between inorganic and organic causation cannot be better illustrated than by contrasting the phenomena which characterize a body while it is alive with the processes that go on in it when it has ceased to live.

In place, therefore, of vague assertions that organic activities have intelligent character, reveal the tendency which Von Baer called *Zielstrebigkeit,* or are mysteriously presided over, according to Hans Driesch, by a "psychoid" which directs without exerting any causal action, we get the account of a process due, under organic conditions, to the impelling and guiding force exerted by the organism as a whole over its own parts, and to the nature of motion as a procession away from conditions of greatest stress towards conditions of equalized stress. We also see the so-called perfecting tendency rationalized and explained by reduction of it to dynamic factors. Not only are the organic units urged into efficient and economic configurations by the resistances they meet with in functioning: the entire organism, by its very nature, is under a differential

stress impelling it as part and whole to higher and more perfect means of self-maintenance, the direction of least stress being the direction which leads to completer division of labor, higher specialization of function, improved forms of reproduction, closer cooperation between members of the species, more complex and finer adjustments to environment. When, therefore, the nutritive and general conditions are favorable, the organism ascends in the scale of life, handing down to its offspring such variations as obtain the support and sanction of the species. We must meanwhile assume that if adaptations in the inorganic do not arise capriciously, there can be no ultimate fortuity in plant and animal. If, in spite of counter currents, there is a vast cosmic drift towards intelligent, enduring forms, the organism cannot be helplessly subjected to the unrule of chance. The variations out of which such forms arise in the interest of life, whether minute and gradual, or saltatory, must be held to acquire their ultimate adaptation, their intelligent character, not fortuitously, but through movement towards definite goals pre-figured by the needs of organic self-maintenance, the character of matter, and the nature of the motion process. And though natural selection may powerfully help to conserve what has been won, the winner of the advantage is always the organism itself.

INTELLIGENCE PLUS CONSCIOUSNESS.

Recognition of the nature of intelligence as fundamentally dynamic relieves us from the presumed necessity of everywhere reading "psychic powers" into nature as a means of comprehending intelligent adaptations. Why, then, should some of these adaptations be implicated with consciousness? How does it come about that in the organic we have two sets of end-reaching movements—those of the internal processes which do not involve consciousness,

and those of the relational activities which do? It is here assumed that in the inorganic system, lacking the reciprocality above described, there can be no psychic character, minute or massive. But in the simplest protoplasmic system the interdependent units are subject as whole and as part to the shock of all the changes imposed upon them by environment and arising within them as means to maintenance. It is not merely that the disturbance is diffused —it calls forth a reflex movement of all the units, and it is this reciprocal tremor of all of them jointly implicated and reacting which constitutes the objective side of whatever awareness or feeling the incipient organism has thus far reached. As with advancing organization uniformities of outgoing action come to be set up, uniformities of action from without provided for, feeling will be more and more withdrawn from established orders of change and restricted to variations or departures from such orders. For motions that recur with constancy, for processes that go on with unbroken regularity, there is no shock and no reaction of the system: with the one and the many interadapted to them, the regularities are taken up into the very process of the time-and-space whole. Consciousness remains only for those muscular and mental activities which, falling outside routine, evading organization, cannot become automatic. And even for these, with the gradual splitting up of function and division of work, it comes finally to be located in the brain.

Consciousness is thus of the sudden, the irregular, the unaccustomed. How do the organic activities undergo division into usual and unusual? Here the conception of group actions and reactions will aid us. Each specialized tissue, physiological process and organ has the character of a class, since each arises out of a special set of class determinations. It is because the properties of matter exist in classes, and because the incident forces exist in

classes, that the organism must utilize or react on these for self-maintenance by means of processes and organs that constitute classes. Gravitation is an objective class, and has its organic correlate in organs of locomotion; ether and air-vibrations form classes, and have their complements in organs of vision and hearing. Out of the incoming of repeatedly iterated determinations that are likes to each other, requiring continually iterated outgoing actions of fundamentally like kind, arises each organ and process through which self-maintenance is secured. But while organic class is thus the correlate of objective class, there are degrees of resemblance in the conditions of action, incoming and outgoing. In the lower life the correspondence between organization and activities is close: this is the realm of motions requiring little or no choice, of mechanical responses, of reflex action, of the so-called tropisms and, to a greater or less extent, of instinct. As complexity increases, as the impulsion to self-maintenance demands more precise and varied adjustments to environment, as motion and manipulation grow more selective and discriminating, the organism analyses out the original likeness of conditions into differences, with the result that the organic class comes to be split into actions which can be automatically performed and other actions which must be voluntarily and consciously performed. The change occurs not in the physiological processes, which are relatively constant, but in the muscular and mental activities which relate the organism to its surroundings. Some of these are not accompanied by consciousness for the reason that the determining elements are likes to each other; others must be consciously performed because, though belonging fundamentally to the class, the elements differ superficially from each other and thus need effort to include them within it.

This breaking up of the original holophrasm of the

class may be illustrated from the sense-perceptions. In vision, for example, there can at first be only the vaguest sense of light and shade; in the latest stages we find these analyzed out into multifarious varieties by the class organ, the shade definitized as objects, the light split up into colors. So sound, at first a generalized sensation, comes at last to be broken up into differences, with a specialized appliance in the ear for recognizing each. Now all the varieties thus arising belong fundamentally to their class —that of light or sound—and are therefore dealt with by their class organ. But as varieties they have to be won, as it were, from their superficial difference into coalescence with the group to which they belong. For the higher organisms, to be alive means to be ever moving, manipulating, mentally active—means to be involved in constant changes of relation to environment through a complex of experiences in which no two muscular movements, no two mental adjustments, no two cognitions of objects, or even of the same object, can be exactly alike. The brain is thus meeting situations which are at once fundamentally old and superficially new. The whole of organic ascent is, in fact, a progressive analysis of reality into those variations from the class which demand effort for their inclusion within the class. It is for these variants from an original theme, this doing and perceiving of things superficially new yet fundamentally old, that the organized brain mechanisms do not suffice. For them special connections have to be made between the neurons, fresh paths furrowed for the flow of nervous energy; and in these deviations from the customary, from the usual, from the organized, the whole brain engaged in the effort quivers in response, and there is not only the deed, but also the illumination of it which we call consciousness.

Has consciousness any causative or determining part in organic processes? If we mean by consciousness only

our awareness of what we are doing, the reply is that it has not. If we mean by consciousness the physical correlate of that awareness in brain and nervous system, the answer must be that it is so in-and-in woven into the physical changes from which it arises as to be an inseparable factor of them. Originally the shock to which feeling is due could not have been more than a sort of by-product of the functioning of organic matter in self-maintenance. In the long, slow evolution of life from lower to higher, and finally to highest stages, it must have been gradually interorganized with the rest of the physiological processes. It has thus become an indispensable part of those processes, not in the sense of determining their occurrence, of being their *raison d'être*, but in the sense of helping to make up a whole of interadapted actions all of which require each, each of which is necessary to all, with the result that specific kinds of psychical change are kept inseparably linked with particular sorts of physical change. That the objective shock should appear to us as consciousness remains inexplicable, as is sufficiently shown by the difficulty of describing awareness in other than terms of itself. The illumination we have as conscious is an irreducible fact, set off by its uniqueness of character from all other facts. All we can say of it is that it subsumes in brain and nervous system a highly specialized form of the same reciprocality of power which the universe itself displays. For all grades of feeling there is needed that relation of the implicated one to the including many, that swift interchange between the dominating many and the subordinated one, which are possible only in living matter and within the narrowly delimited areas of organic systems. If on the objective side consciousness is the attitude of the system towards unusual and unorganized changes within it, on the subjective side it is such changes seen in the many-faceted mirror for

which each organic unit, reflecting and reflected, is also at once thing and image of thing to all the organic units.

In consciousness, as in everything else pertaining to life, we thus come back to the organism wielding itself, in possession of itself—to a system arising out of nature, belonging to nature, yet none the less set off from nature by organic causation and the activities of self-maintenance. It is only because our psychical states are conspicuous, the physical states which produce them concealed, that we have been eager to promote consciousness to causal rank over that which originates it; only because the obvious teleology of our voluntary activities, muscular and mental, shuts us out from the teleology of the process which underlies them, that we have clung obstinately to the view which makes consciousness the principle of intelligence and the driving power of life. Little as we can scrutinize the physical correlates of our conscious states, we are bound to assume that all these states without exception, from the simplest feeling of discomfort to the highest flights of abstract thought, are founded in non-psychical processes, and fit these as the cast fits the mould. The set which we have towards life and life's activities is plainly imposed, not primordially by the feelings we have as knowers, but primordially by the physical body we inherit and wield as doers. As the real source of organic teleology is not a psychical principle, but the nature of the organism plus the motion process, so the real guide to intelligent adaptations is not consciousness, but the ends implicit and demanding realization in a system under impulsion to maintain itself. Finding consciousness associated only with some of the organic activities and absent from the whole of the inorganic world, we are compelled to regard it as a peculiar and narrowly delimited aspect of reality, and at the same time a superficial, not a fundamental aspect of life. Finding teleology in both organism and universe,

we are compelled to trace it not to a psychical, but to a dynamic principle. Consciousness suffers nothing by being delivered from the embarrassment of irrational claims. It rises in dignity the more we can view it as the outcome of a process deeper than its own; it becomes more precious to us the more closely we can link it with life; it takes highest rank when, as revealer, it patterns forth, however imperfectly and unsatisfactorily, something of the wonderful resources of the organism and the inexhaustible richness of cosmic power.

CHANGE AND ENDURANCE.

Power is here regarded, not as an aggregated multiplicity, but as a divisionalized unity. Its differentiation into material units makes groupings, collocations, moving systems and configurations of them possible. The impulsion of the cosmic system to self-maintenance, its conversion of differential stresses through motion into equalized stresses, result in the arrangements we call intelligent adaptations. The "end" of maintenance, of endurance, is implicit in the nature of the system, whether inorganic or organic; it finds realization through the displacements and replacements which the system imposes. Contrivance or design in nature is the elementary form of the end-reaching we witness and practise in our own lives as organisms. Out of a process which has endurance for its goal emerges in both living and not-living the intelligence of "intelligent adaptations." In accepting this view we deliver purposiveness in nature from its supposed origin in psychic elements; the notion of a cosmos ruled by mind widens out into the thought of a cosmos potential of mind, as well as of possibilities unutterably beyond the grasp of mind; the appeal to the argument from design yields to the argument from the nature of power. Purposiveness in life is meanwhile emancipated from both vitalistic and

mechanistic assumptions—from the crude symbolism of the one, from the narrow empiricism of the other. The organism is under determination: in all its acting and developing the cause-effect relation may be traced. But as a self-maintainer it is none the less relatively self-centered, self-acting, autonomous. The physico-chemical properties of matter are servants, not masters in the house of life. The determinism is that of a system that wields itself. The freedom involved is that of organic causation.

The cosmic flux, like the organic flux, must be regarded as a means to endurance. Change is eternal, but things flow and continue to flow in the same general way, in modes that can be relied on and predicted: the vicissitudes succeed each other, their order remains. There are plenty of unteleological conflicts in nature, yet the emergence of enduring form—there as suns, planets and their furnishings, here as the crowded display of the organic world—shows beyond peradventure a final triumph of the collectivizing over the dissipating, the purposive over the fortuitous tendencies. As motion in the inorganic ensures the conservation of power, so changes in the organism are means to self-maintenance. The functions of life in the moneron and in man do not essentially differ; the class frameworks remain throughout organic ascent—it is only their content which undergoes multiplication and enrichment. In human activities, in tools and machines, even in social and political systems, men work towards the things which endure. Speech is a body of signs whose general character survives all changes; human thought, through whatsoever vicissitudes and controversies, keeps its fundamental elements unmodified. There is an inertia of mind as well as of matter, a conservation of ideas as well as of energy. The drift of science is towards a body of truths that shall endure; sure foothold in their own affirmations is also the quest of the theologians. The world's

great thinkers still "beacon from the abode where the eternal are."

Insistence on the flow of things is thus only half the story. All material objects are time-and-space wholes: only as such, and as arising out of the cosmic whole, can we have reliable knowledge of them. To sum them up as present existences in time or as local existences in space is to cut them off from the universe. Even if we could isolate the flow from the things that flow, we should need to take account of its direction and end. Only by realizing the flux as teleological, as a working towards intelligent forms, do we grasp the meaning of change, and to that extent the meaning of reality. The changes that seem to expend power do but insure its continuance. The power that changes not survives all change as the very possibility of change. The French poet spoke more wisely than he knew when he declared that "nothing endures eternally, in order that all may endure."

EDMUND NOBLE.

BOSTON, MASS.

DEATH.

O DEATH, in thee we reach life's consummation;
In thee we shall find peace; in thee our worries,
Anxieties and struggles will be past.
Thou art our truest friend! Thou holdest
The anodyne which cureth every ill.

Thou lookest stern, O Death; the living fear thee;
Thy countenance is severe and awe-inspiring,
And creatures shrink from thee as their worst foe.
They know thee not, for they believe that thou
Takest delight in agonies and horrors,
Disease and pain. The host of all these ills
Precedes thee often, but thou brook'st them not.
'Tis life that is replete with suffering,
Not thou, O refuge of the unfortunate,
For thou com'st as surcease of pain; thou grantest
Release from torture, and thy sweetest boon
Is peace eternal. So I call thee friend
And will proclaim thy gift as greatest blessing.

Death is the twin of birth: he blotteth out
The past but to provide for life's renewal.
All life on earth is one continuous flow
Which death and birth cut up in single lives
Of individual existences
So as to keep life ever new and fresh.

Oblivious of the day, that moulded us,
We enter life with virgin expectations;
Traditions are we of parental past,
And gain of our expanding souls we hand
To the succeeding ages which we build.
The lives of predecessors live in us
And we continue in the race to come.
Thus in the Eleusinean Mysteries
A burning torch was passed from hand to hand,
And every hand was needed in the chain
To keep the holy flame aglow—the symbol
Of spirit-life, of higher aspirations.

'Tis not desirable to eke out life
Into eternity, world without end.
Far better 't is to live in fresh renewals,
Far better to remain within time's limits.
Our fate 't is to be born, to act our part
And, whether we approve of it or not,
When all is finished, to depart resigned.
Again and evermore again, life starteth
In each new birth a fresh new consciousness
With wider tasks, new quickened interests,
And with life's worn-out problems all renewed.
But we must work the work while it is day,
For thou, O Death, wilt hush life's turbulence
And then the night will come to stay our work.

When we have tasted of the zests of life,
Have breathed the air of comprehension, have
Enjoyed the pleasures of accomplishment,
When we have felt the glow of happiness,
The thrill of love, of friendship, of endeavor,
When we have borne the day's heat and have sweated
Under the burden of our tasks, we shall,

Wearied of life's long drudgery, be glad
To sink into the arms of sleep, to rest
From all our labors, while our work lives on.
As at the end of day we greet the night,
So we grow tired of duties, pains and joys
And gladly quaff the draught of Lethe's cup.

Wilt thou be kind to me, O Death, then spare me
The time to do my duties and to finish
My lifework ere I die. Let me accomplish
The most important tasks that lie before me,
So when I die I have not lived in vain.
But has my purpose grown beyond myself,
I shall be satisfied and welcome thee.

Kinder thou art than thou appearest, Death!
Peace-bringer, healer of life's malady,
Thou lullest us into unconsciousness.
Thine eye, well do I know it, solves the transient
Into mere dust; but thou discriminatest,
Thou provest all, O just and unbribed judge,
Appli'st the touchstone of eternal worth
And thou preservest the enduring gold.
Thou settest free the slave, soothest all anguish,
Grantest an amnesty for trespasses,
Abolishest responsibilities,
Bringest cessation of the troubles which
Are harassing in life. Thou simply closest
A chapter in time's interesting book,
There to remain as we have written it,
And so thou dost no harm. Happy is he
Who neither feareth nor inviteth thee.

I honor thee, great sanctifier Death,
Lord of the realm of no return—High Priest

Of the unchangeable, thou consecratest
Our souls when gathering them unto their fathers
In their eternal home; I honor thee,
Yet will not seek thee! I am here to live
And will abide until the hour arrive
To enter on my Sabbath eve of life.
But neither shall I shrink from thee, for truly
I see no cause why I should face thee not.
Thou dost not doom me to annihilation,
Thou wipest out my trace of life as little
As any deed can ever be annulled.
Indeed, thou comest to immortalize,
To finish, to complete, to consummate,
To sanctify what I have been and done.
Therefore, I shall be ready at thy call
And deem the common destiny of all
Meet for myself, so when thou beckonest,
Friend Death, grant me thy sweet enduring rest.

EDITOR.

CRITICISMS AND DISCUSSIONS.

A SATIRE ON THE PRINCIPLE OF RELATIVITY.

EDITORIAL INTRODUCTION.

The principle of relativity has been advanced by two physicists, Dr. Albert Einstein, of Zurich, and H. A. Lorentz of Leyden; and by a mathematician, Hermann Minkowski of Göttingen. The two former are still living, but unfortunately Professor Minkowski has died prematurely. Their theories have found much recognition and the number of their admirers in the scientific world of to-day is legion.

The results of the new view are most astounding. The traditional conceptions of science are abolished and the new doctrine that replaces them is so paradoxical as to justify the boldest dreams of the superrational and superscientific fancies of savage and medieval mysticism. The public stand in breathless astonishment and the man of common sense is baffled.

This new movement has been discussed editorially in several articles in *The Monist*,[1] which have recently been collected in book form under the title *The Principle of Relativity in the Light of the Philosophy of Science,* and in these a rather critical attitude is taken toward the bold conclusions which the new school draws from what seems to us a neglect of a proper recognition of the principle of relativity. The present number of *The Monist* contains an article by Professor More of Cincinnati University which criticizes certain details of the theories of the leading relativists, taking them of course quite seriously. But there has now appeared on the scene a Vienna engineer, one Mr. Leo Gilbert, a satirist who pours upon the heads of the inventors of the new physics,

[1] "The Principle of Relativity," April, 1912; "The Philosophy of Relativity," October, 1912; "The Principle of Relativity as a Phase in the Development of Science," July, 1913.

and also upon their most prominent supporters, the vials of his sarcasm in a book which bears the vigorous title *Das Relativitätsprinzip, die jüngste Modenarrheit der Wissenschaft.*[2] He dedicates the book to Mr. Rudolf Goldscheid, joint editor with Prof. Wilhelm Ostwald of the *Monistische Jahrhundert.*

The irony which pervades this little book is certainly of sufficient interest to justify extended extracts which will prove both instructive and amusing. The advocates of the new principle of relativity will say that our author harps too much on the same string, viz., the paradox of inverting time, allowing the future to precede the present and even the past and thereby rendering it possible that an effect may precede its cause. We will even grant that he does not, at least in this book before us, try to do full justice to the intentions of his adversaries; but a satirist has the privilege of a poet and we need not take him over-seriously. Even a greater man than Leo Gilbert, the Attic playwright Aristophanes, plied the whip on the wrong man, on Socrates, when he meant the then modern school of Sophists, the relativists and pragmatists of decadent Athens. So we must forgive Mr. Gilbert for sometimes being too severe, and also for sometimes hitting too hard. The book is thus described in

THE PUBLISHERS' ANNOUNCEMENT.

In the last seven years the principle of relativity has found wide circulation and has gained many adherents. Nevertheless many have also been puzzled by the strange logical contradictions of this new theory which make heavy demands upon their credulity. Philosophers, as in fact all educated people, are perplexed by the fact that absolutely new views of the nature of time are introduced and all previous habits of thought seem to be overthrown. At the same time it must not be forgotten that such new principles are always thought out only by the few and that the majority must be reconciled either against their will or submissively to the views which are thrust upon them by these few. Confident opponents of the principle dare not venture forth, for they are at once repulsed, overwhelmed with scorn or otherwise subjected to unpleasant experiences. But when an entirely new cosmogony is to be constructed then its opponents must be heard first, since they contribute most

[2] Verlag Dr. W. Breitenbach at Brackwede i. W., Germany. The extracts quoted in this review have been translated for *The Monist* by Lydia G. Robinson.

of all to the clarification of the new system of ideas. Therefore an author should be gratefully received who has found the extraordinary courage and love of work carefully to weigh this miracle-working hypothesis of the "relativity of time" and to publish his opposition in a satirical form which is universally intelligible and at the same time affords entertainment. As Leo Gilbert himself says, it grieved him exceedingly that he was compelled to a satirical treatment of the matter by the indolence of the specialists and their skill in killing by silence. In his preceding work on "The Foundations of Exact Science" (*Fundamente des exakten Wissens*), he had shown that it is the seriously constructive, positive work in science which he has at heart. Even in this satire he has not been driven so much towards negation as toward positive results, for the most noticeable thing in his satirical treatment is the fundamental reality which led him to the solution of a problem that has been hanging in the balance for sixty years, namely the Fizeau experiment. Hence we must say that the entertaining form in which Leo Gilbert's polemic is carried on, while providing an important explanation of the doubtful aspects of the principle of relativity, at the same time affords captivating reading—though naturally only in so far as the reader is not too zealous an adherent of the principle of relativity prone to condemn in advance every other view as heretical. However one may feel about the book, in any case it is an interesting document of an intellectual movement and a scientific controversy of our own day.

THE MEANING OF RELATIVITY.

[Leo Gilbert thus complains that relativists neglect to define the term:]

These investigators have neglected first to determine what they wish to understand by "relativity." They have used the innocent word blindly, regardless of consequences. The designation of the new "principle of relativity," which contains not the slightest trace of "relativity," has arisen in the same way. Foolish misuse of words!. . . .

It is an ancient and atrocious evil of science that it uses the same word interchangeably in the most different meanings. Since the mind must each time bring order anew into this chaos, mental operations become difficult and many errors arise. It thus becomes a strict requirement of the economy of thought that these gentlemen should define relativity before they apply it. Nevertheless I can

inform them right now that such a definition is rather difficult and troublesome, for in every relative there lurks an absolute, and in every absolute a relative. Thus in the relativity of Newton, the absolute appears in the guise of the specific zero of the relative. For instance, if only one of the velocities $b, d,$ and e equals zero the absolute values of all the others are at once known to us. Thus, if the sun had the velocity $b=0$ in space, then the absolute velocities of all our planets would be known. Here I find another fundamental error from which the relativists proceed and which throws light upon their remarkable philosophical comprehension. They think, as Schames expresses it, that nature must remain "faithful" to itself everywhere; but these gentlemen mean by this that nature must remain faithful in every case to their own lack of intelligence.

The fact is they believe that everything, even time itself, must be made relative in the sense of the relativity of space. These scholars have no suspicion that the world and the knowledge of the world rest upon contrasts, that there is an absolute for every relative. If science designates certain features of space as relativity it must still strive to determine (1) what are the limitations of this relativity, (2) where it passes into its opposite, (3) what we must regard as its counterpart, its correlate, the absolute. Doubtless space and time are opposed to each other as the infinitely great arch-integral to the infinitely small arch-differential, as the boundlessly extended to the exactly limited, as the constantly being to the disappearing functional, as the relative to the absolute....

The greatest mistake of the gentlemen is that they ignore a self-evident fact in order to discover a monstrosity.

NEWTON TURNS IN HIS GRAVE.

[The author begins his satire with the following introduction:]

Lately I dreamed of meeting the immortal Newton. "How do you come here?" I asked in astonishment. He answered:

"I am an old gentleman, over two centuries old and quite stiff and lazy; so I would not have thought it possible that I could turn over in my grave with such lightning rapidity, but it gave me a dreadful shock when Professor Sommerfeld of Munich recently declared at a learned congress that the fabulous new principle of relativity of Messrs. Lorentz, Einstein and Minkowski had been well established for six years. I decided to come back to earth at once and find out what was the matter, and how these over-zealous people had supplemented my old theory of relativity by

a wonderful new abracadabra charm. My spirit glided with youthful agility into the body of a professor who is in the habit of looking down upon his colleagues in a high and mighty way. Then I betook myself to a number of famous old scholars of the highest rank who kept outside the crowd, hoping that I might find out something from them. But here I was no less astonished. These geniuses, like all the neutral professors, privat-docents, young freshlings and mathematical dilletantes, all gave me one and the same answer. Whenever I attacked the fiction, its sponsor would defend the 'new ideas' like a lion. He would quote formulas, roots, velocities of light, experiments of Fizeau, Michelson, Morley for perhaps ten minutes at a stretch. Then right in the heat of the debate he would grow strangely lukewarm, would become lame in the hips and finally creep off into an impenetrable thicket of brambles, saying: 'Oh I have not been able to enter into the matter in such detail, I only tell what I have read. But still there seems to be something deep in it; that may be taken for granted. Just think of the Kantian ideality of space and time and how it is again confirmed here!' Hardly would the lion have uttered the word 'Kant,' of whose meaning of course he had barely the remotest suspicion, when he would again become a ravenous beast in his safe bramble thicket snarling at me and showing his teeth."

"What the Devil's the use of being 'specialists,'" cried Newton greatly excited, "when they have 'only read,' when they have not even taken the trouble to study carefully these wonderfully fascinating 'problems' and 'new discoveries'? Why are they blind believers in miracles, idly boasting of Kant's ideality without knowing what it is? And yet they pretend to be men of exact science! Are they not ashamed of their intellectual beggary and vagueness?"....

When I awoke and the master had disappeared, I reflected upon my dream and thought of the many friends who apply to me, mostly philosophers and naturalists who are "not mathematicians," people who in their guileless honesty believe that the sacred image of Sais is hidden behind the tomfoolery of formulas in whose brambles the originators of the "new principle" are hopelessly lost. They ask: "Can you not explain to me what there really is in it? It must be very magnificent!" Now, my dear inquiring friends, I will tell you!

TWISTED LOGIC.

The logic with which these gentlemen treat science will be best illumined by the mental lightning which Professor Einstein permits to flash on us only "incidentally," quite aside from the other fundamental attainments of his nimble intellect. He carries out a calculation with his formulas of relativity and goes on to say:[3] "This result signifies that we must regard as possible a transmissive mechanism by means of which the anticipated effect precedes the cause (perhaps accompanied by an act of will). Although in my opinion this result in a purely logical sense contains no contradiction, nevertheless it is so absolutely opposed to the character of our entire experience that through it...."

Isn't that great! The very top notch of freedom from prejudice! According to Professor Einstein no logical contradiction is involved when the effect takes place *before* the cause, hence when, so to speak, the effect acts as the cause of the cause, in which case then the cause would appear as the effect of the effect. How droll! And why not? Let us illustrate: The farmer's wife finds an egg in the stable. In a week she will go to market and buy the hen that is to lay the egg she has already found. Who can object to this? Einstein agrees with it.

An oak has been cut down; a chauffeur and three passengers of an auto lie with broken heads by the side of the road. Afterwards, however, it occurs to the chauffeur to put on the highest speed and rush against the oak, lose his presence of mind and let go the steering wheel. The coroner of course held the inquest over the four bodies two weeks before the accident, and three years previously they were buried—the effect before the cause.

We also admire the education attained by a good old man whose parents are only just born—all of which Professor Einstein would find quite "natural."

These are the same gentlemen who have been so busily and fashionably engaged with entropy, and who when it comes to the point, do not even know that their "entropy" means that every occurrence has a direction in time.

These gentlemen fire before they load, post up laws of nature before they discover them and place knowledge in their head when

[3] In the *Annalen der Physik,* edited by Wien and Planck, Leipsic, 1907, Vol. XXIII, pp. 381-382, in the article "On the Inertia of Energy Demanded by the Principle of Relativity."

they have no head. Nothing is impossible for the true scholar—*den echten Gelehrten, den verkehrten, unerhörten, überquerten, reingeleerten—quasi geistesgestörten.*

Einstein may be a distinguished scholar in the matter of figures and formulas, but when it comes to the philosophy of nature and the theory of cognition he is without any doubt an eminently useless man, a genius of illogic. And the rest are no better.

Equally wonderful is Prof. H. A. Lorentz's theory of electricity. According to his view matter consists of matter—of course worthless dead matter—but with electrons added to it. Nature takes electrons and attaches them to simple motionless matter. ("Synthetikon glues, sticks, cements everything: one tube only twenty pfennigs!") Now if we tickle both matter and electrons, matter alone will suffer it patiently. The electrons however become excited, and then they are electrical. On the other hand, if matter is given a velocity it no longer remains indifferent; it straightway changes the unchangeable, its mass! For a long time Lorentz proposed that it only changed its length, but then other insisted that it changed its mass. In the world of "these" physicists who are also called "new," everything is conceivable. Where guns kill before they go off and go off before they are loaded, "mass" can easily be sweated out through the pores of matter. This is not at all surprising in a physics according to which the electrons break their way, or "stream," through a massive copper wire with a velocity of 300,000 kilometers.

Why? How? By what means do electrons cling to matter? What is the use of matter if it is inactive? What is the deeper connection, the real significance of these things? What is the sense of "mass" if mass be changeable and if it can disappear? How can the law of conservation suddenly be surrendered? What after all is the meaning of this "new" physics which denotes an absurdity radically lacking in system, a ridiculous tomfoolery?

What is mass after all?

Just consider the grand unitary world-conception, so actual, so sensible, so beautiful and unbroken which the grand old thinkers have built up, men like Faraday, Hertz, Mach, Ernst Haeckel and others, and compare with it this crazy, giddy, confused whimsicality. It presses the wine before the grapes ripen and cuts the grapes before they are grown; before men drink they stagger and reel in drunkenness and folly.

HOLES IN THE ETHER.

Every year about ten thousand professors of high schools, universities, technical institutes, and also some independent scholars devise about ten thousand queer hypotheses of which they themselves are the only fanatical adherents.

Perhaps ten or twenty of them find a dozen blindly faithful believers among their boon companions at the tavern. One will whisper to another with lifted brows: "O that Bookmiller, he has a head on him! The world will hear from him yet. He has proved positively that if a camel be only large enough it can go through the eye of a needle! He has proved it by higher mathematics!"

Higher mathematics! That chokes off every contradiction! A few out of the hundreds of thousands are more successful. They find believers by the help of a high-sounding name. This was the case a hundred years ago with phlogiston, and fifty years ago with the world's entropy-end and with countless other jokes. Such is the case to-day with the "new" principle of relativity. According to it the largest camels have actually passed through the tiniest needle's eye of this principle. It is the eternal victory of banality operating with "higher mathematics"....

Poincaré, whose calling as a mathematician is not attacked here, in his lecture on "The New Mechanics" arrives at this remarkable decision: "We can almost say that there is no longer matter but only holes in the ether, and in so far as these holes seem to play an active part it consists in the inability of these holes to change their location without influencing the surrounding ether which exerts a reactive influence on such changes."

This entire statement is fabulous, yea insane, and might pass in a sanitarium for incurables. Fabulous is the typical academic caution with which the famous mathematician expresses himself. He does not say, "Thus it is and not otherwise." He only says, "We may *almost* say." Nothing rises up within him against the paradox that holes play an "active" part. He does not perceive in the least that just this thing we can not say, not even "almost." He expresses himself very cautiously with professorial diplomacy: "in so far as these holes 'seem' to play an active part." What genuinely scholarly delicacy: "seem"! We can assure M. Poincaré with absolute certainty that this time appearances are most lamen-

tably deceptive; that holes can disclose no activity; that they lack all fitness to evoke any sort of "reaction." We could assure M. Poincaré, the famous mathematician, (if he were still alive) that by holes is understood the opposite of anything that can parade as energy or as capacity for work. By "holes" the human intellect has always understood and will continue to understand the unsubstantial, the unenergetic....

When certain gentlemen speak of "nature philosophy" they remind me of the Italian "nature singers," who certainly do not understand singing and do not even possess voices; but that is exactly what they call nature.

THE ELEMENT OF MYSTERY IN THE "NEW RELATIVITY."

First of all we shall try to make the nature of this "fact" comprehensible to the popular mind by a comparison.

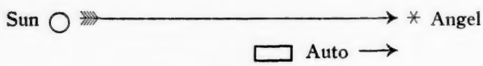

Fig. 1.

Let us imagine an angel flying away from the sun with the velocity of $c = 300{,}000$ kilometers per second. If we stand on the sun we can observe that the angel is moving away from us a distance of 300,000 kilometers every second. This is his velocity with reference to us as inhabitants of the sun.

Let us now place a chauffeur in an automobile which starts from the sun at the same time and rushes along by the side of the angel with a velocity of $a = 299{,}999$ kilometers. Then the angel will gain only one kilometer over the auto in a second. The angel moves away from the chauffeur only one kilometer each second. Hence with respect to the chaffeur the angel possesses a velocity of only one kilometer per second. This is clear and positive.

Now what would we think of a man who came along and said: "No that is not right. Instead, the fact is that there continues to be a difference of velocity of 300,000 kilometers between the angel and the auto. No matter what the velocity of the auto, the angel precedes it always 300,000 kilometers more every second; for instance 3,000,000 kilometers in ten seconds. In other words, no matter how rapidly the chauffeur may be moving, the velocity of the angel with reference to him is exactly as great as it is for the stationary inhabitants of the sun." Would we not judge that such a

man was not quite normal in the upper story, or think with Goethe that

> "A contradiction absolute
> Is always for the wise, no less than fools, a mystery"?

A COMMON-SENSE EXPLANATION OF THE PRINCIPLE.

Before we enter upon a critique of the principle of relativity, we shall here present in brief outlines what it really signifies. It treats of the motion of bodies in space, and of the velocity of transmission of rays of light. And indeed we observe the motion of bodies which travel equal distances in equal times and which therefore possess so-called "translatory" velocity. We assume with the relativists that the ray of light is transmitted in an absolutely stationary medium, in cosmic ether or in empty space, hence independently of the motion of any kind of body.

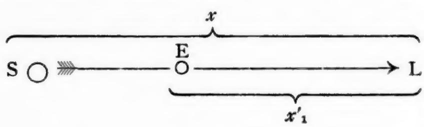

Fig. 2.

At first glance the matter seems supremely simple. Let S represent the sun. Let S——L be a ray of light that travels through empty space with a velocity of $c = 300{,}000$ kilometers per second. Let E be the earth which, as we arbitrarily assume, moves with a velocity of $a = 100{,}000$ kilometers in the direction of the arrow. Every one can now say: If we stand on the earth and measure the velocity of the ray of light it is only E——L $= c - a = 300{,}000 - 100{,}000 = 200{,}000$ kilometers ahead of the earth at the end of one second. In other words, the more rapidly the earth moves, the smaller is the difference between the two velocities. If the earth were to dash through the universe as rapidly as the ray of light, then the latter would possess no difference in velocity with reference to the earth; i. e., they would both arrive at the same time. The ray of light would possess a velocity of 0 with reference to the earth. This is what would be expected clearly and unequivocally from the standpoint of common sense. But this is the point where the relativists violate logic and replace the sensible conclusion by a very different one. They say: Whatever may be the velocity a of the earth, the ray of light will always outdistance it by $c =$

300,000 kilometers. If the earth stands still the ray of light will have a velocity of 300,000 kilometers measured from the earth. But if the earth moves at the rate of 100,000 kilometers still the light ray will exceed it every second at the rate of 300,000 kilometers. This is Einstein's famous "principle of the constant velocity of light."

The reader sees at once that this last assertion is impossible. Its conclusion is contradictory to logic if, as above assumed, we accept the hypothesis of the relativists according to which a ray of light is entirely independent and is supposed to move independently in an ideally conceived empty, absolutely stationary cosmic space. In this space the ray of light could have a velocity only $c - a = 200{,}000$ kilometers greater than that of the earth. Hence the view of the relativists contains a fallacy. They would have the relative velocity of light with respect to the earth $c - a = c$, however great a may be, and this is an impossibility. On the basis of their hypothesis, $c - a = c$ can only be true when $a = 0$; i. e., if the earth remained as motionless as the sun. How the relativists come to their remarkable claim will be discussed elsewhere. Here we will merely mention briefly that they lean upon an experiment made by Michelson and Morley. From this they think they may deduce that the ray of light always possesses the same velocity of transmission of 300,000 kilometers with respect to the earth, no matter with what velocity and in what direction the earth may move.

This conclusion they call "the fact" which is identical with their "principle of the constancy of the velocity of light." Upon this "fact" they erect the principle of relativity. The principle of the constant velocity of light may be expressed very generally as follows:

Fig. 3.

Let A be a body from which a ray of light proceeds and B another body which receives the light, e. g., a mirror or the eye of an observer. Then this principle will mean that the velocity of transmission of light with respect to A and also to B always possesses the same value c no matter what the velocity of each of these bodies.

THE MICHELSON EXPERIMENT.

Whenever one speaks about the principle of relativity to serious estimable people, minds of the first rank, they reply: "To be sure I haven't dug into this principle very deeply as yet, but there must be something in it. There is a fact at the bottom of this principle. Michelson has performed an experiment that...."

Fact? My head swims! Have we then come to this after a century of exact research that we talk about "facts" like bad lawyers? Some bewigged head peeps through a telescope and sees a black elephant whirling around between Jupiter and Saturn. Hence it is a "fact" that black elephants fly back and forth between the planets! What! Do you doubt it? Still, it is a fact! Professor Valerian Eyeblinker saw it himself in the observatory! Unobjectionable!

What is the real fact about this "fact"? Only that Mr. Valerian has seen something black! Whether it is an elephant, or whether it moved among the constellations, or whether it was a fly in the telescope, or a *mouche volante* in his own eye, is no longer a fact but the interpretation of a fact.

Of course there are no "facts" but only phenomena and their interpretation. For instance it has always been a "fact" that the earth turned around its axis and around the sun, but for thousands of years the phenomenon was given the interpretation that at night the sun slipped under the earth in a stove-pipe and hid there until it came out again in the morning to wander across the firmament. When Galilei and Kepler found other interpretations people would not believe at first that they were right. For thousands of years every one had been able to see the sun rise in the morning, even children; it was a "fact." And yet it was not a fact. Then what is a fact?

To the relativists the ray of light constitutes an object of "fact." Since by its nature this is just as unknown as the angels of religious myths, anything can be invented about it that one wishes.

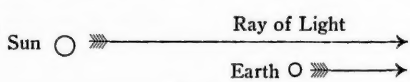

Fig. 4.

Hence, a ray of light moves away from the sun at the rate of 300,000 kilometers a second. According to the statement of the

astronomers the earth possesses a velocity with respect to the sun of barely 30 kilometers a second, hence $\frac{1}{10000}$ of that of light. If the relativists' hypothesis is correct, that the ray moves independently in a stationary vacuous cosmic space, then we must regard the velocity of light with respect to the earth in the direction of the earth's motion as $c - a = 299{,}970$. In the direction perpendicular to this there would be no subtraction. The ray would retain its $c = 300{,}000$ kilometers per second. Therefore it must be confirmed practically by experiment whether a ray of light in the direction of the earth's motion shows a lesser velocity than a ray perpendicular to this direction.

Michelson has performed a corresponding experiment with a ray from a lamp and has found that a ray of light in the direction of the earth's motion by no means remains 30 kilometers behind a ray perpendicular to this direction, but that both rays travel equal distances in equal times....

Thus the result of Michelson's experiment is—as Hertz has long surmised—very simple and a matter of course. The ray of light depends only on the atmosphere through which it moves. It is an undulation of the atmosphere.

THE FOUR RELATIVIST POSTULATES.

An intelligent and versatile author, Herr Fritz Müller, published an excellent popular presentation of the principle of relativity entitled "Der Zeitgeist," in the *Berliner Tageblatt* of October 16, 1911, following upon a lecture of Professor Einstein in Zurich. In order to facilitate the reader's insight into this subject, we here quote his expositions. The comments do not come from Herr Müller but originate with the present author.

A short time ago Professor Einstein gave a lecture on the relativity of time before the society of naturalists in Zurich. When he had finished, a wave of excitement passed over the learned heads. A fallacy had been pointed out, a fallacy as old as humanity itself. For hundreds of thousands of years the apparent experiences of our organs have deceived us into thinking that there is an absolute time in cosmic space. Both the child and the critical scholar believed it, but the assertion is false. Einstein, on the basis of the preliminary studies of other physicists, has corrected it and found that there is no such thing as absolute time. Time is dependent on motion in space.

I know that at first sight this statement does not arouse interest.

It leaves the layman as cool to-day, as did the discovery that the earth turned around the sun, and not the sun around the earth, in Galilei's time. Galilei was an intellectual revolutionist when he exposed an optical illusion and deduced the inexorable consequences which created anew for mankind the image of the universe. Einstein exposed a fallacy in the conception of time, and what are the consequences as regards our world-conception? Here they are:

1. There is no rigid time. Time shrinks together with motion in space.
2. There are no rigid bodies. Their forms flow with motion in space.
3. Space and time are interchangeable.
4. There is no ether.

The principle of relativity is decisive for these remarkable results. What is relativity? The fact that there is nothing absolute in the universe. Every condition depends upon another. Taken mentally, philosophically, this has been clear to us for a long time.... That velocity in space and that time as the content of consciousness are relative is a truism, but the physicist stretches his notion of the term "relative" still farther.

VELOCITY EATS TIME.

Suppose two clocks keeping the same time to have each an observer and to stand side by side. Now suppose one of them with its observer suddenly rushes out into space with the velocity of light. The two observers have previously agreed to telegraph the time every second by means of a light signal. Men have no other medium but light by which to communicate with each other about the simultaneousness of two occurrences in different places. Further let us bear in mind that the criterion of simultaneousness is that the passage of the ray of light should take as long in one direction as in the other. Is this the case with the two clocks when they stand side by side? Obviously yes. And if one clock is traveling with the velocity of light? Obviously no; for the stationary observer, it is true, receives at regular intervals of one second the stipulated light signal from the other, but the latter does not receive any from the former. Whenever the stationary observer wishes to signal his time to the moving observer—whether in a fraction of a second after the start, or one, two or three seconds after—he is never able to do so, and the other one waits in vain for a signal. The signal behind the moving observer will never, never in all

eternity, overtake him if he travel with the same velocity as the signal itself. Hence the criterion is wrong. After one, after two, after three seconds the clocks would show a difference of one, two or three seconds. But in the judgment of the stationary observer this would indicate that the moving clock would be just that much too slow. In our extreme case in which the journey is made with the swiftness of light the stationary observer would believe that the other clock would not arrive in time at all; that there time would stand still. The Einstein equations actually arrive at this result. The same is true, Einstein says, of the observer traveling with the clock, namely, that in the opinion of the stationary one he would never grow old. "And what if he return to his point of departure on a broken line?" somebody asked the lecturer in the discussion. "Then in our judgment he would remain as young as when he started," answered Einstein in all seriousness, "even if we who remain behind had in the meantime become gray-haired; the equations furnish for every direction of motion, even for motion in a broken line, the same results without variation." We look at each other. That sounds fabulous. Fabulous? Certainly, the old fables of the monk of Heisterbach, of Rip van Winkle, of Urashima Taro come to our minds. It is strange how popular imagination has taken the same direction with the Germans, the Americans and the Japanese—all three tales concern people whose life stands still for many years, while those around them age. Thus at their return they find another land and a different generation.

"And if we imagine," another one objects, "that there was some sort of an effect which could be transmitted along a cord, we will say, with a velocity greater than that of light?" "Then it would have to be possible," answered the lecturer, "to devise a mechanism by means of which an effect could be produced upon the past." "And the result would be?" "Merely that this idea is so contrary to our experience that we are compelled to reject it until the opposite is proved. Hence we must assume on the basis of our previous experience that a velocity exceeding that of light is impossible, that it is absurd." Again the hearers are reminded of something, this time it is the remarkable novel *The Time Machine* by the Englishman Wells, who a dozen years ago made his engineer hero construct a machine by means of which he could set himself back into the past. In a remarkable anticipation of future research that poet speaks of time as of a fourth dimension which is of equal validity with our ordinary three dimensions of space, indeed is

even interchangeable with them. Nevertheless the mathematician Minkowski, building upon Einstein's foundation, comes also to the conclusion that physical occurrences are represented in a four-dimensional space in which time plays the same rôle as the three physical dimensions. Further Minkowski concludes: "From this hour space-in-itself and time-in-itself are to disappear entirely into shadows and only a sort of union of the two will retain independence, for no one has observed a place except at some time, or a time except in some place." And time? It may be entirely or partly replaced, extinguished by motion. That which we call time will be wholly compensated by a motion which proceeds with the velocity of light. A body traveling in space with the velocity of light will, from our point of view, be forever timeless. Hence, space is time and time is space. To put it strongly, motion eats time.

[A strange proposition! If a motion equals the velocity of light which is our means of communicating the time of a distant clock to us, time disappears and so we learn that velocity eats the time. Time is annihilated.]

TIME OVERTAKEN BY VELOCITY.

Let us consider a case which may also be found in somewhat altered form in the work of Camille Flammarion.

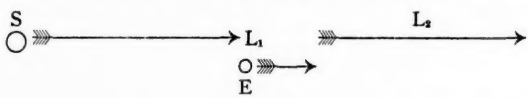

Fig. 5.

Let us imagine the earth E traveling before the sun with a velocity greater than that of light. Then the rays L_1 which come afterwards will never reach it but will always be farther and farther behind. We shall not see the sun any more at our left. But yesterday and the day before yesterday, and many hundred years before that, rays L_2 have been emanating from the sun which have already proceeded far ahead in cosmic space. The earth now travels into this ocean of rays and catches up with all of them in succession, since it of course travels faster than a ray. What will happen now? Our eyes will be pierced successively by the rays of light which the sun sent forth yesterday, ten years ago, one hundred years ago. Hence we will obtain pictures from the sun's past in earlier and earlier years, will observe its life and

development inversely, will witness the sun-child as boy, as baby, as an infant in swaddling clothes, until the sun is decomposed into the rotating masses of vapor from which it has originated. We would witness the entire process of the sun's growth in an inverse direction....

Our eyes travel with the earth along the rays L_2. Hence these rays reach our retina from the right and we shall no longer see the sun on our left, for in that direction it has disappeared from our view. From this time we shall perceive its reflection only from the right, accordingly in the direction opposite to that in which it really is....

These mystifications, these optical illusions in the observation of things were known to us long before the relativists. The relativists attempted merely to adopt this idea, but they have corrupted it instead by dragging their formulas into it, in consequence of which the visions here described count for them as "facts." In the case just described, in which according to the old conception we would have to look back into the past of the sun, all values even become imaginary; that is to say, we lose the ground of reality entirely from under our feet. The physical experiment becomes an absurdity. Even the course of our clock becomes imaginary, yes, even time *in propria persona,* space, all processes, our organism, mankind,—in short the disease becomes incurable.

OBJECTS SHORTENED BY VELOCITY.

When the famous physicist of Leyden, Prof. H. A. Lorentz, advanced his incredible formula for an impossible behavior of light towards moving bodies, reality got beyond his control. But since his was a genuine Faust nature, it was easy enough for him to construct a handful of brilliant new laws of nature. The mathematical genius of Leyden created a "new" law for himself and a "new" world. Boldly and prudently he advanced a proposition which contradicts the law of inertia so laboriously attained: *Bodies which move through space with a velocity are diminished in the direction of their velocity,* computed only in the direction of their motion. What a phenomenal labor it required to diminish an earthly body, for instance bars of porphyry and steel, only $1/200000$ of its length, Lorentz did not take into consideration. He worked a charm contrary to the dictum of all renowned conjurers who must know better, namely, that legerdemain is not magic.

That such a shortening—it is called after its gifted originator a Lorentz-contraction—nevertheless must obviously possess a most profound energetic significance, that it operates in its consequences upon the surrounding universe to the most remote stars, that it must transform our entire world-conception—these points the master has wisely passed over in silence.

To be sure this wonderful law would be of great practical value. For instance it would no longer be necessary to send fat people to Marienbad to partake of its bubbling waters and to torment themselves with constitutionals by the hour. They would need simply to stand daily for an hour at a time with their bodies in the direction of the earth's motion. Because of the earth's velocity their bodies must needs experience Lorentz-contractions and become thinner, and this by means of the power of the beneficent new laws of nature.

Yes, let us say it with ruthless plainness. We must not let the earth too greatly exceed its alleged velocity of 30 kilometers, nor must we allow her ever so to forget herself as to assume a velocity of 300,000 kilometers, for then it would be all over with us. The Lorentz-contractions would no longer be child's play. We would become quite flat. Indeed flat is no name for it! No, we would be infinitely thinner than flat; for since the dimension of the earth would contract to nothing—absolutely nothing—in the direction of its motion, then in this extreme case it would become only a circular disk like the paper hoops which the circus clown holds before the leaping equestrienne on her dapple gray horse. Only this hoop would be infinitely superior in strength to the flattened earth, for it is of the tangible thickness of paper, whereas the thickness of the earth would be 0, nothing. All dimensions of the earth in other directions would remain the same in their original extension. Hence it would have merely a circumference, it would be a flat disk without thickness, a bodiless body, a breath! A scheme, a concept! So much can a planet be reduced when it assumes velocities which approach the velocity of light—and when at the same time it falls into the hands of famous mathematicians....

But what if the velocity of the earth exceeded even that of light? If it traveled at a rate of 350,000 or 400,000 kilometers per second, which of course might actually be so, though if it were we would not be able to find it out. Would it then have a minus thickness? What would a minus thickness look like? No, according to the formula it attains an *imaginary* thickness, but it always

continues to be *really* broad, high and round; only in its thickness, (i. e., in the direction of the motion) is it *imaginary*.

That would indeed be a sublime thought! By spinning out this idea I could, if I thought it necessary, obtain not only the friendship but also the admiration of the relativity physicists who are to-day in the lead. They would erect bronze monuments to me while still in the flesh.

[However, there are some differences of opinion. Leo Gilbert continues in the next chapter, saying that everything is mere appearance. He says:]

The fat men have congratulated themselves too soon. Unfortunately the Lorentz-contractions are ineffective. For, sorry to say, Prof. K. A. Einstein in Zurich, the most thoughtful of the mathematical prestidigitators, has deprived us of this new benefaction, and so has rescued Marienbad from desolation. Let no one who stands with his body in the direction of the earth's velocity be persuaded that it really is reduced. The reduction is only apparent. But in appearance, in imagination, the phenomenon positively takes place!

MINKOWSKI ON TIME.

[Formerly we thought that time is absolute in the sense that the present moment is everywhere. It is the ever-present Now which is in the same way here and there, on earth, on the sun, on the star Sirius and on the most distant sun in the universe. But Minkowski teaches us something better and grander; he says time is relative. Mr. Gilbert continues:]

It is asking altogether too much of the ordinary human intellect, but there can no longer be any doubt about it,—now is no longer now, day before yesterday is no longer day before yesterday. But yesterday is to-morrow, the future is past, and to-day is nevermore! The relativity of time! Delightful word! Intoxicating term! If it only had some sense! The late Hermann Minkowski is said to have been a very eminent, some say from hearsay an "ingenious," mathematician. This is sufficient to account for the high respect which his colleagues render to a talent in which Minkowski was absolutely lacking, namely the interpretation of natural phenomena. He lacked every suggestion of a trace of ability to touch creatively the philosophy of nature or the theory of cognition. A professor of mechanics in a technical high school in Germany once recommended to me the ingenious lecture of Minkowski on

time and space, and added, "which unfortunately is hard to understand." A funny recommendation of something which can not be easily understood, hence may even be false. I believe indeed that the lecture is obscure enough for many professors of mathematics, and it is only from this haziness that we can understand that the gentlemen of the eightieth congress of naturalists at Cologne dissolved in admiration before the splendid intellect of Minkowski. The uncomprehended has always been the greatest religious mystery before which mankind has bowed. If you wish to be successful be obscure, be unintelligible. Especially for naturalists! particularly for exact scientists! and most of all for mathematicians!....

I at once wrote back in reply to the excellent professor of mechanics that unfortunately the lecture was only too intelligible to me since I had been engaged on the subject for twenty years, and that I had perceived nothing in it more clearly than a certain childish guilelessness with which Minkowski had passed by the little that had already been said about time and space by better thinkers....

[If he had studied into the matter] it would have been impossible for him to have combined all the fabulous absurdities which could find comprehension in that wonderfully constructed brain, in which a medley of numbers, physics and Kant, dance around in confusion.... Minkowski tells us: "From now on space-in-itself and time-in-itself sink entirely into shadows and only a sort of union of the two can retain independent existence." Of course this childish poetry about the "sinking shadows" which brings to mind school memories of Homer and Dante is wonderfully affecting to the hearts of mathematical physicists, so that many hasten to quote this remarkable phrase of "a sort of union" in their publications.

Moreover Minkowski has made the most surprising discovery that time is not the same time everywhere. We poor inferior people had hitherto believed that the present moment which we now experience on earth is identical with the present moment in the most remote finitenesses. In short, that time is absolute, rigorously absolute, as rigorous as the most rigid of all the laws of nature.

Oh, fudge! Minkowski and Einstein now inform us that even time is relative. When our clock upon the earth strikes twelve, on another star in another system of planets it may be striking three quarters of an hour and five seconds after twenty-six, or even thirteen hundred and seven. The simultaneousness of events, that

powerful controlling law which alone makes possible a thought, a comparison, a demonstration, a knowledge—the law of the absolute simultaneousness of the infinitely small differential of the moment which divides the past from the future by a hair, and which we call the present—this iron law of reason is simply abolished by an overrational professor in the midst of the acclamations of other professors.

"No hour strikes for the happy man"—especially at the beer table of the naturalist congress when the gentlemen have done joyful justice to the enlightening beverage....

In their liberality with senseless contentions the relativists are downright spendthrifts. One reads in Minkowski the subtle clairvoyant assertion that there are several cosmic spaces. "Several cosmic spaces!" This remarkable mode of thought and speech has already found its quotation fiends, for there is no folly which does not arouse some scholar, or author, or journalist, by the fascination which lurks in paradoxes, to dish up the affair scalding hot. "Several cosmic spaces!!!" How very amusing!! What can separate them from each other? Cardboard walls? Or strong leaded glass? Probably plate glass, because otherwise the ray of light upon which the whole relativity bluff is based could not shine through from one cosmic space to the other. Or perhaps the partition walls are made of thickened ether? Or baked electricity? Or dried formulas and watered scholars' prattle?

Among the people who have fallen into the trap of the Messrs. Relativists with particular *élan* and *chic* is to be mentioned Geheimrat Dr. Max Planck, professor at the University of Berlin. In an essay which appeared in the *Umschau* of October 29, 1910, and which bore the pompous title, "The Position of Modern Physics with Relation to Phenomenal Nature," even he quotes admiringly Minkowski's bold phrase [with reference to time and space] about the shadows and the union. He finds that the principle of relativity leads "to a very far-reaching—one might almost say revolutionary—consequence with reference to the conception of time," "since a proposition with regard to time does not contain a physical sense until the observer's state of velocity to which it refers is taken into account...."

Referring to Minkowski, Planck closes: "Accordingly the physical world accessible to our observations possesses four equally justified interchangeable dimensions, three of which we call space and the fourth, time."

EDITORIAL CONCLUSION.

Much may be said in reply to Mr. Leo Gilbert's satire; how here and there he slightly twists the propositions of the inventors of the principle of relativity, and how in his exuberant humor he draws consequences which they would not countenance; indeed some of them are actually disavowed. Relativists will probably remind him that according to their claim the velocity of light is the greatest possible velocity, which would cut out some of the best and most humorous comments he makes. Yet we must grant him that no valid reason for this assertion of a truly absolutistic nature has as yet been produced. We feel tempted to make further comments as to what may be said in favor of the new doctrine of the principle of relativity and the half-truths it contains, but we prefer to give the relativists an opportunity to take the stand themselves, although they will probably meet this new attack with complaints that they have been misunderstood and that it is not worth their while to given an answer. To kill with silence (or as the Germans say *Totschweigen*) is the safest mode of defence for a weak cause. Mr. Gilbert claims that this is invariably the method of the relativists, and so for his motto he places on the title page of his book the conjugation of the verb, which he calls the "chorus of the craft":

> "Ich schweige tot,
> Du schweigst tot,
> Er schweigt tot,
> Wir schweigen tot,
> Ihr schweigt tot,
> Sie schweigen tot."

In brief we do not endorse all the conclusions of Mr. Gilbert's criticisms, but we deem his book worth a perusal and a careful reply. The mere humor of it will richly repay the reader for the hour spent thereon. P. C.

EDUARD STUDY'S REALISTIC WORLD-CONCEPTION.

Eduard Study, professor of mathematics in Bonn, has published an essay on "The Realistic World-Conception and the Theory of Space."[1] The task before him is thus formulated on page 62:

"In what consists the epistemological value of the geometrical

[1] *Die realistische Weltansicht und die Lehre vom Raume.* Brunswick. F. Vieweg & Sohn, 1914.

systems comprised under 'non-Euclidean geometry'? May we regard them as hypotheses of equal or approximately equal value with the Euclidean in respect to the structure of our space? And if so, may we content ourselves with these hypotheses, or shall we not be obliged to make other cautious attempts to approximate the unknown reality, or rather the image of this reality? And is the path of natural science indeed the only one which can procure for us at least some sort of explanation of this natural geometry? Yea, is it indeed a practicable route?"

Our author mainly follows Helmholtz, and by "natural geometry" we must understand the results of natural science, especially physics, and the results of these investigations concerning space-relations.

Professor Study proposes four theories of space: 1*a*, spherical geometry; 1*b*, elliptic geometry; 2, pseudo-spherical geometry; and 3, Euclidean geometry which may also be called parabolic geometry. As his conclusion he makes four propositions (pp. 111-112):

"1. However natural geometry may be constructed, no distinction can be made, or only with great difficulty, between it and any other of the four designated kinds of measurement-geometry (*Massgeometrie*) by the aid of our present expedients.

"2. The four kinds of geometry above mentioned are to be preferred among all conceivable hypotheses.

"3. The Euclidean in turn is preliminarily to be preferred among these four hypotheses, and it will continue to be valid where the distances concerned are not too great.

"4. It would be rash to regard as settled the epistemological question of the structure of empirical space.

"By means of their at least very close relation to experience, all four kinds of geometry now possess a particular interest among the infinite multiplicity of geometrical systems which the fancy of mathematicians, 'the mathematical play-instinct,' may contrive. It may be convenient to have one word in common for these four particularly important geometrical systems. We take the liberty to interepret a term coined by Helmholtz in this sense: we shall call the above-mentioned four systems of three-dimensional geometry 'systems of physical geometry,' *cum grano salis* in the systems of abstract geometry applicable to physics."

The book contains incidentally many thoughtful remarks, and is mainly useful for its critical observations, especially of Poincaré's views and also those of Mach. We here quote in translation a few

passages on positivism and pragmatism as of special interest (pp. 41-51):

"Indeed the positivists have done an excellent work. What mathematician or physicist would not have derived the richest instruction from Mach's *Mechanics*,[2] that splendid book which cannot be studied carefully and frequently enough, which at each perusal exerts anew its stimulating and refreshing influence! But there is very little in it which a thoughtful realist might not also have said or subscribed to.

"But the 'principle' with which alone we here have to do is criticism and nothing but criticism, pure and indeed downright barren negation, a kind of guillotine or a Procrustean bed which is always too short. The history of philosophy says that it was originally an act of liberation from the excesses of a wild speculation. Perhaps, but why then the exaggeration? To-day we miss in the management of this mechanism the very spirit in whose name it accomplishes, or tries to accomplish, its destructive activity, the spirit of criticism. The 'anti-metaphysical' and 'anti-dogmatic tendency' has itself become an unruly dogma; 'philosophical decrees' are set in opposition to philosophical decrees. It seems to be a universal law that no idea is safe from unlimited exaggeration. Witchcraft, belief in ghosts, spiritism, fourth dimension of space, latticed molecules, ions and electrons, generally speaking matter in the usual sense of the word, apriorism, causality, energies, chemical theories, and I know not how many more—this quite varied company is mercilessly put to death and disappears in the same wholesale grave which is destined for metaphysics and its whole witches' sabbath of near and distant relatives as an unadorned tomb. Death levels all. By its side stands the preacher of positivism who has been ordained for such occasions, and he delivers a funeral sermon which though not exactly pious still is certainly imposing. To the assembled mourners and curious spectators—we had almost said guests—he brings the comforting and exalting message that they are witnesses of an event of cosmic importance. In this moment is dawning a new epoch of civilization, the molecular, etc., superstition is destroyed, the beautiful era of science free from hypotheses and an energetic 'culturology' has begun. Then all go home to the accompaniment of music. One man who stands somewhat aside allowed his thoughts to roam. Liberty,

[2] English translation by T. J. MacCormack. Chicago, Open Court Publishing Company.

equality, and fraternity came to his mind, and scenes and faces awoke memories within him. It had happened not long before, and it had been almost as beautiful as now. All the more important 'riddles of the universe' had been solved, and at that time they had buried spirit which had become superfluous together with its own witches' sabbath. Some of the personages now being buried were then cruelly slaughtered and lowered in the grave, of course in their character as the offspring of spirit. And then also had dawned a new cycle of civilization stretching on into eternity, the era of a loftier materialistic civilization. How rapidly the civilizations replace each other even to-day! That is progress indeed! And already another funeral train is approaching with the escort of a corresponding civilization, and in the distance another and behind that still another. Pragmatism, activism, intuitionism, or however else the newest mysticism of the *élan vital* may be called, are drawing near, X-ism, Y-ism, Ism, Ism, Ism! Experts even prophesy that Hegelianism will arise once more. At any rate it is a joy to live! Who would trouble his head over questions which so rapidly lose all attraction....

"We shall now taste of the sweetest newly-ripened fruit of the tree of knowledge, pragmatism. As early as in Mach we find 'the principle of the economy of thought' repeatedly mentioned. This is at bottom the old practical demand—universally recognized, for instance in mathematics—that the results of science be presented as simply as possible. This is now made into a main principle of cognition and the foundation of all science. Hypotheses which as such must be rejected by the positivists are justified by its aid. It really contains a sound kernel, for the cases have been many from time immemorial where the decision between corresponding hypotheses in the natural sciences has been so made that the simpler was given the preference. But the establishment of the theory that the simplest hypotheses are always the best has turned out so inadequately with Mach that one of his opponents proposed the formulation: It saves thought to assume that the thought which saves thought is the correct one....

"The critical point of the affair is apparent in a more general application which Mach has already given to the same thought. It is biologically advantageous (for the profit of humanity) to make the simplest hypotheses....

"The American and English pragmatists Dewey, James and F. C. S. Schiller have developed this conclusion derived from the

theory of biological advantage into the form of a so-called theory of truth. We can notice here, and not unjustly, a terminological misuse. The above mentioned authors might equally well deny the existence of truth altogether. However, on what it depends and what particularly concerns us here is the assertion of the subjective character of all scientific criteria whether or not they are called the criteria of truth: and thus far pragmatism is really a consequence of the theory of biological advantage, and indeed its necessary consequence, if this theory can be said to have any comprehensible meaning at all.

"According to James the individual must balance pleasure and pain against each other, and in so doing he attains a conception of truth, or a criterion of truth (for between the two there is no distinction in pragmatism) and this criterion is valid for himself and is binding on no other person.

"As James himself says, the theories of other pragmatists though differing somewhat in form are essentially in agreement on this point....

"Truth, according to Dewey, is 'that which gives satisfaction' which is obviously a very pleasant conception of truth....

"According to the third representative of pragmatism, F. C. S. Schiller, 'that which works' (following James) is true. By this bewilderingly simple formula the old problem is definitely solved; the 'childish fancy' that there can be a truth independent of ourselves is set aside. But what works to-day does not need to work to-morrow. (*Acqua passata non macina più*). 'An idea is true so long as belief in it is useful for our life' (James). Hence we have here the biological advantage as a utility for 'our' life and at the same time the correct insight that the scientific criteria deducible from it (the so-called criteria of truth) must vary according to time and circumstances. Truths, or what the pragmatist calls truths, come, continue and pass like living creatures; the truths themselves, not our knowledge of a (realistic) truth. But since the pragmatist recognizes and allows only his own conception of truth we must rejoice that, for instance, the Pythagorean proposition still seems to some extent to be useful, otherwise we ought to abolish it on the spot!

"Thus 'does pragmatism loosen all our theories. It has indeed no prejudices, no dogmas that block the way, no strict canon for the demonstrative force of arguments.' It does not 'cling' to logic, and therefore (if we are permitted still to draw logical conclusions)

on proper occasions goodnaturedly would let two and two be equal to five. Therefore 'it possesses a great advantage in the religious domain.'

"This 'thoroughly joyous philosophy' commends itself particularly to large circles of people from the fact that it 'extends the realm in which man can seek God,' and that it is 'democratic.' Indeed it seems to have been copied from the form of government of the United States: a philosophy of the people, for the people, by the people. Let every one come and join....

"To be sure it has already been known that the ideals of the old world culture are 'childish fancy' and that the hitherto so-called science which paid no heed to utility can for the most part be thrown on the rubbish heap. But that such a gospel could be proclaimed on this side of the Atlantic (Europe) is a new idea and a particularly painful one.

"It is conceivable that such things need not be said twice to the 'pragmatically so successful' daily press, especially in the land of unlimited possibilities. Who will doubt that a theory of truth which dispenses on demand, not only milk, butter and cheese but 'religious consolation' as well, would not awaken interest and applause everywhere, even among the laity? Even Herr Ostwald who is inspired by pragmatism (as might be expected) reports how stirringly the lectures of James have affected the whole(?) of intellectual New York....

"To be sure, that the 'bread-and-butter standpoint' corresponds exactly to pragmatistic principles, the pragmatists are not at all aware. James even speaks of shameless slander. By our citations we have placed the reader in a position to judge for himself.... No, the cause of pragmatism's ill luck lies in pragmatism itself, in its principle, and in the jelly-like consistency of its jelly-fish philosophy which dissolves under close scrutiny.

"Pragmatism, which appeals to every one, ought to take account also of the dark side of human nature. This theory which pretends to be a philosophy of life must pay particular heed to its probable effects, to its own 'pragmatic results.' Is there any horrible deed which could not be and has not been proved to be 'pragmatically successful?' Not only people of low 'ideals,' but a conquistador or high inquisitor might have appealed to the pragmatic principles if he had the good fortune to have lived at a time when this philosophy was in existence. 'Nothing is true and everything is allowable.' This is the device which pragmatism ought to choose if it

would keep its consequences in sight. But the pragmatist does not possess a mirror, and, more than that, like the goddess of justice he has a bandage on his eyes. Who can be blamed for not taking seriously philosophers whose wisdom (*inter alia*), is equivalent to saying that there are as many and more kinds of 'science' than there are religious sects!

"Still if a critique regardless of consequences is desired, we need only to be reminded that in every-day life there are also uncomfortable and even quite dreadful truths, truths which are none the less truths for all that; granite truths which can neither be shaken nor interpreted away, and against which all pragmatic balances and dialectic feats are powerless."

The strength of the author lies apparently in his critical powers. His positive suggestions will probably not be considered as helpful, yet the book is suggestive and pleasant reading throughout.

P. C.

RECENT PERIODICALS.

The number of *Scientia* (*Rivista di Scienza*) for September, 1913, begins with an article by Aldo Mieli on the theory of substances with the pre-Socratic—this would be better described as pre-Aristotelian—Greeks. The first part, which is here published, contains an examination of those theories which were intended to explain the unceasing transformations of substances in nature, from the earliest speculations to Empedocles and including him. At the time of Empedocles's appearance, the problem was: all transformations being deceptive appearances of the senses, to seek what is true and stable behind all these false appearances. The solution of the problem, curiously enough, is obtained for the greater part by the adoption of a doctrine which had failed in geometry, the doctrine of the Pythagoreans (as Paul Tannery and others have shown) that geometrical figures are sums of points. This doctrine was partly destroyed by the discovery of incommensurable lengths and completely by the "triumphant" reasoning of Zeno. Future Greek mathematicians built up their splendid edifice of geometry on new and more secure foundations, but the fundamental conception of the theory of composition out of points passed into the theory of the composition of bodies. Three directions then presented themselves: (1) Empedocles's theory, which admitted of a limited number of primitive substances that are invariables and, by their mix-

ture, make very different substances appear; (2) the theory of Anaxagoras, which considered all the different substances as existing from the beginning and independently of one another, and made the appearance of that which predominates depend on phenomena of association and dissociation of similar elements; (3) the theory of Leucippus and Democritus, which postulated one and one only original element and explained the various aspects which it presents by supposing that the particles or "atoms" of it are susceptible of receiving different forms or having different relative positions. The fact is emphasized, as Burnet has done in his *Early Greek Philosophy,* that these theories are connected with Paramenides's idea of the invariability of the true "being": atoms were the "being" of Parmenides. The doctrine of Empedocles is examined here in some detail. It was the one most generally adopted by the ancients, by the philosophers of the middle ages, and even by the theoretical chemistry of the nineteenth century. The article is followed by another in the next number (see below) of *Scientia,* and the whole question will be discussed more fully in the first part of a forthcoming book on the general history of scientific thought.

Thomas Moreux, in an article discussing the question as to where our sun is dragging us, gives a useful sketch of the development of researches on the proper motion of the fixed stars from the earliest times up to quite modern ones. A. J. Herbertson has a geographical essay on "The Higher Units," the theme of which is to prove that "regional leviathans" ("if the geographical region is a macro-organism, then men are its nerve-cells") exist and that we each are a part of one.

Eugenio Rignano presents the second part of his studies on the evolution of reasoning, which is concerned with the progress from intuition to deduction. In the first part of this article, the author has said that parallel to, and in consequence of, the passage from concrete forms to forms more and more abstract, reasoning acquires an increasing and more extensive capacity of application, which, starting from quite simple intuitions, carries it to the most complicated deductive processes of science. It is with this aspect of the evolution of reasoning that this part is concerned. A further article dealing with the higher forms of reasoning is promised.

Sigmund Freud deals with the interest of psycho-analysis for psychology. Psycho-analysis is a medical method which tries to cure certain forms of nervous disease (*Neurosen*) by psychological

technic. This article explains, by a series of examples, what the author has claimed, in a book published in 1910, for the new science.

H. Jacobi, in an article with a title asking what Sanskrit is, studies the importance and the position of Sanskrit in the evolution of language and Indian civilization. S. Jankelevitch has a critical note on the thinking horses of Elberfeld. In the department of prehistoric ethnology, Sigmund Feist gives a general review on the question of the country where the Indo-Europeans originated. There are reviews of books and periodicals, a chronicle of events in the scientific world, and a supplement containing French translations of the German, English, and Italian articles.

The first article in the number of *Scientia* for November, 1913, is the second and final part of Aldo Mieli's account of the theories of substances with the pre-Socratic Greeks. In the first part, he had shown that the early doctrines were suddenly abandoned, and three currents of thought—represented by Empedocles, Anaxagoras and Democritus and the other atomists, respectively—were formed. This part deals with the two last currents. It is very interesting to see that the doctrine of Anaxagoras, which has been quite misunderstood until of late years, is capable of taking, when developed, a quasi-modern form (see the author's article in *Isis,* November, 1913; cf. the notice below).

J. C. Kapteyn, of the University of Groningen, writes an article in English "On the Structure of the Universe," in which he considers the questions as to what the discovery of what is known as "star-streaming" has done and what it promises to do for the solution of the problems of (1) the distances of the stars from one another in the line of sight; (2) the history, that is, the evolution, of the stellar system. "It has been my aim," says the author, "to show *not* that much has been done, but that there is a beginning: *not* that we have entered far into the promised land...., but that a few pathways are being mapped out, along which we may direct a hopeful attack."

Werner Mecklenburg shows how the theory of electrolytic solutions affords an example of how two currents of investigation, which seemed quite independent of one another (Pfeffer, Raoult), were united in a higher synthesis (Van't Hoff).

Sigmund Freud, in the second part of an article continued from the preceding number of *Scientia,* deals with the interest of psycho-analysis for the non-psychological sciences, comprising the sciences of language, philosophy, biology, the history of evolution,

the history of civilization, the fine arts, sociology, and pedagogy. Charles Guignebert, in the first part of an article on the dogma of the Trinity, discusses the primitive triads and the baptismal formula, and shows, on a particular example, how the fundamental beliefs of a religion originate, develop, are fixed, weaken, and die. A. van Gennep has a critical note on the lacunae of modern ethnography.

There are two general reviews: one a biological one by H. Piéron on the evolution of modern scientific opinion on the question of mimicry; and the other by W. Oualid, which is an annual review of economy. There are the usual book reviews, review of reviews, chronicle, and supplement containing French translations of the German, Italian, and English articles.

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The second number of *Isis* appeared in August, 1913, and began with a short note by the editor (George Sarton) on the aim of *Isis*. *Isis* is the only review in which the methodological, sociological, and philosophical points of view are associated with the purely historical point of view; this association is effected because it is only by making all these points of view and all these methods converge that historical researches acquire their full signification. Still, history, though an indispensable instrument, is a means and not an end: the end is the philosophy of the sciences. David Eugene Smith, owing to the fact that we have recently come into possession of a quantity of new material, is able to give, in an interesting article on "The Geometry of the Hindus," a presentation of some of the salient features of the Hindu geometry more clearly than has heretofore been done. Antonio Favaro has a short article on the *Carmen de ponderibus* of Guarino Veronese. There are three articles dealing with genius and heredity: one in German by Wilhelm Ostwald, one in English by W. C. D. and C. D. Whetham, and one in French by George Sarton. The second is on "Three English Men of Science"—Newton, Darwin, and Kelvin, and contains the rather surprising statement: "To each one of these men, either continuously or at some period of his existence, religion and not science seemed the end that was best worth pursuing." The third is to be continued, and is on the question of how to increase the intellectual efficiency of humanity. The present instalment contains an introduction and sections on scientific genius and genius and the race. The rest of the number is devoted to a useful chronicle of events and undertakings, analyses of books and articles, and a

continuation of the admirable bibliography of recent works on the history of all the sciences.

The third number of *Isis* appeared in November, 1913, and contains seven articles, three in Italian, two in German, one in French, and one in English. Antonio Favaro writes on the well-known mathematician Niccolo Tartaglia (1499-1557) and the printing of some of his works, particularly the *Travagliata Inventione*. Julius Ruska writes on mineralogy in Arabic literature. Icilio Guareschi writes on Ascania Sobrero (1812-1888), the discoverer of nitro-glycerine, the centenary of whose birth took place last October. Agnes Arber discusses the botanical philosophy of Guy de la Brosse, who died in 1641, and was the founder and first superintendent of the Jardin des plantes of Paris. Aldo Mieli writes on the development and utilization by modern chemistry of the ancient theory of the transformation of substances of Anaxagoras, as distinguished from those of Empedocles and of the atomists. Ernst Bloch studies the ancient atomic theory in the modern history of chemistry. Finally, the editor, George Sarton, continues his article on how to increase the intellectual efficiency of humanity; the present instalment contains discussions of heredity, the inheritance of intellectual aptitudes, and environment and heredity. There is a portrait of Denis Diderot, the bi-centenary of whose birth took place last October. Various notes and reviews of books are given, and the valuable bibliography of works on the history of all the sciences is continued.

The number of *Isis* for February, 1914, concludes the first volume of this new and very interesting periodical. The editor, George Sarton, writes on the present tendencies of the history of mathematics, and begins by distinguishing between what may be called the pure historians and those—philosophers, pedagogues, or sociologists—whose aim is to *utilize* the history of mathematics for other ends. The object of the review *Isis* is that of the latter class. Then the author proceeds to discuss Léon Brunschvicg's *Les étapes de la philosophie mathématique* and Pierre Boutroux's *Principes de l'analyse mathématique*, both of which are analyzed at great length by Emile Turrière later on in the number. Though the author is very well-disposed towards both books, he finds himself obliged to conclude that neither of them realizes a complete historical and critical synthesis of mathematics. Ernst Bloch has a long and valuable article on chemical theories with Descartes and the Cartesians, and the article deals principally with Descartes,

Mayow, Lémery, Guillaume Homberg, Johann Bernoulli, Steven Blankaart, and Georg Ernst Stahl. Then follows Gino Loria's stimulating address to the International Congress of Historical Studies at London in 1913 on the mathematical glories of Great Britain—Bede, Alcuin, Abelard of Bath, Roger Bacon, John Peckham, John Holywood, Thomas Bradwardine, Richard of Wallingford, John Maudith, Cuthbert Tonstall, James Gregory, Robert Recorde, William Oughtred, John Napier, Thomas Harriot, Henry Briggs, Henry Savile, Isaac Barrow, John Wallis, Lord Brouncker, John Caswell, William Neil, Christopher Wren, Kenelm Digby, John Pell, Isaac Newton, John Craig, James Stirling, Patrick Murdoch, Abraham de Moivre, Roger Cotes, Brook Taylor, Edmund Halley, Robert Simson, Samuel Horsley, William Wales, Colin Maclaurin, William Braikenridge, Edward Waring, John Wilson, James Ivory, and many names of more modern mathematicians. There are rather many mistakes in spelling, but the list is otherwise strikingly accurate. Waldemar Deonna writes on Jacques Goffarel, a precursor of the present theory of the origins of art. Philip E. B. Jourdain has a long and rather technical article on "The Origin of Cauchy's Conceptions of a Definite Integral and of the Continuity of a Function," in which the part played by Fourier is emphasized. There are various notes and reviews, a continuation of the admirable bibliography of all the publications relating to the history and organization of science, and an index to the first volume. We hope to see many more volumes of a periodical which shows a worthy carrying out of a lofty ideal. Φ